Using informative Bayesian priors and on-farm experimentation to predict optimal site-specific nitrogen rates

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Abstract.

Most U.S. Corn Belt states now recommend the Maximum Return to Nitrogen (MRTN) method for determining optimal nitrogen rates, which is based on on-farm yield response to nitrogen trials. The MRTN method summarizes trials for a region of a state. This study combines Illinois MRTN data, Bayesian methods, and on-farm experimentation to provide site-specific nitrogen recommendations. On-farm trials are now being used to provide the information necessary for site-specific management. Recommendations from only a few years of data, however, can be very noisy. One problem is that the needed models to use as Bayesian priors have not been estimated. This research fills this gap. Utilizing data from the Maximum Return to Nitrogen database, Bayesian estimation is used to estimate production functions that have a time trend to account for increased corn yields over time. The estimated models are then used as an informative prior for yield response estimations using on-farm experimental data. Three years of on-farm experimental data from a single field were used to estimate a spatially varying coefficient

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model. The model was estimated using two years of data to predict the third year. The predicted spatial variability was small and uncorrelated with spatial variability in the third year. Even though experimentation did not help with variable rate recommendations, it could potentially help provide uniform rate recommendations for a specific field.

Keywords.
Bayesian Kriging; corn; precision agriculture; spatially varying coefficients

Introduction

The technology to apply variable rate nitrogen is included on the newer equipment used by many custom applicators. Without an accurate method of estimating optimal nitrogen rates, however, it is questionable that variable rate nitrogen application offers much value to producers. Recent research finds that variable rate nitrogen is not yet unambiguously profitable (Biermacher et al. 2009; Boyer et al., 2011; Stefanini et al., 2015; Larson et al., 2020; Queiroz et al., 2023). The current methods of estimating site-specific optimal nitrogen rates are not good enough for widespread adoption of variable rate nitrogen. This research proposes and tests a method to more accurately estimate variable-rate optimal nitrogen rates.

The maximum return to nitrogen (MRTN) approach is used to recommend nitrogen rates in many U.S. Corn Belt states (Nafziger, 2018). MRTN rates are not site-specific (Illinois has three regions) and so cannot guide variable rate application. Other studies used on-farm trials to acquire information necessary for site-specific management (Bullock et al., 2020; Hegedus et al. 2023). The Data Intensive Farm Management (DIFM) project is a collaboration between researchers and producers that uses on-farm experiments to base farm input management decisions (Bullock et al., 2019). When these on-farm trials are used with only a few years of data, they produce noisy nitrogen rate recommendations. Combining the extensive MRTN dataset as a prior to be used with experimental on-farm data could reduce the uncertainty and improve the accuracy of optimal nitrogen rate estimates using on-farm experiments.

Combining MRTN and DIFM data is a Bayesian problem. The current MRTN is in a format, however, that has limited usefulness as a prior. The nitrogen rate calculator can produce a histogram of economically optimal N rates, but what is needed is the distribution of parameter estimates. Illinois MRTN estimations are made using different functional forms for yield response. The most common is the quadratic plateau model, estimated for each site year. Linear, quadratic, and constant functions are used when they better fit the data than the quadratic plateau. The Illinois MRTN usually uses the most recent 15 years of data to estimate regional nitrogen rates. MRTN does not include a time trend when estimating optimal nitrogen rates. Here, the MRTN data were used to estimate a stochastic linear plateau model with a time trend and then used to provide the priors needed for Bayesian estimation using data from an on-farm experiment.

The functional form used to estimate yield response functions can lead to very different nitrogen recommendations. Many models are based on von Liebig's Law of the Minimum, which states crop yield is determined by the most limiting essential nutrient. Recent research has used stochastic linear plateau functions. Tembo et al. (2008) let the plateau vary stochastically from year to year, while Makowski & Wallach (2002) let all parameters vary. MRTN lets all parameters vary by site year. The model used here only lets the plateau vary by year due to the limited on-farm experimental data. Stochastic linear plateau production functions have been applied to wheat (Brorsen & Richter, 2012), winter rye (Tumusiime et al., 2011), cotton (Brorsen, 2013) and corn (Lambert & Cho, 2022; Bouer et al., 2013; Villacis et al. 2020)

One way to estimate spatially varying coefficient production functions is geographically weighted regression (GWR) (Evans et al., 2020; Trevisan et al., 2020; Lambert & Cho, 2022). GWR is often fits data well. Wheeler and Calder (2007) showed that a Bayesian regression model with spatially varying coefficients provided more accurate parameter estimates than GWR. Finley (2011) also concluded that the Bayesian spatially varying coefficient model had a smaller prediction mean squared error. The drawback of Bayesian methods is that they are computationally slow in comparison to GWR. An informative Bayesian prior can improve the accuracy and speed of estimation while also making inference possible. This research goes beyond previous research by using an informative Bayesian prior to estimate a spatially varying plateau model. Estimates are then used to determine optimal nitrogen rates.

Bayesian methods have long been used to estimate yield response functions (Holloway & Paris, 2002; Ouedraogo & Brorsen, 2018; Moeltner et al., 2021; Park et al. 2024). In a simulation study, Lawrence et al. (2015) used Bayesian methods to update the parameters each year, but did not begin with an informative prior. Bullock et al. (2020) stated the greatest value of the on-farm precision experiment came from prior information collected from two previous trials. This information helped producers more accurately estimate a field's optimal uniform application rate. Franz et al. (2020) concluded that among spatial and temporal variables, including soil types, topography, and crop condition, the best predictor of crop yield was historical yield maps. This past research suggests that Bayesian methods are a promising way to reduce the noise in estimates when using only a few years of on-farm experimental data.

The methods developed and demonstrated here are intended as a step forward toward designing a system that will be adopted by custom fertilizer applicators. More specifically, the MRTN dataset is used to estimate a stochastic linear plateau function that serves as a prior for a spatially varying coefficient model that uses on-farm experimental data to estimate nitrogen rate recommendations.

Data and Methods

Maximum return to nitrogen (MRTN)

The MRTN method is a regional approach for estimating corn nitrogen rates for U.S. Midwest states. For example, for Illinois in 2021 the method used 720 corn yield response to nitrogen trials from 15 years, (2006 to 2020), of data (709 trials in 2024, but more than 15 years). These trials were conducted with spring application, sidedress application, or split between a preplant application and a sidedress application of nitrogen. No sites were irrigated. In Illinois, MRTN uses mostly quadratic response plateau (QRP) models (82.7% in 2024), some quadratic models (15%), a few linear responses (2.3%), and zero no-response models (in 2024) were estimated for each site year. QRP models are often used because they fit the data well. The MRTN approach uses a grid search procedure to determine the optimal level of nitrogen. The dataset used here includes some older data that are not currently being used by MRTN. Also, the 2010 to 2012 data used in the MRTN were not obtained. The data used here represents only a portion of the MRTN data. Note that MRTN weights each site year equally. Since more data are available from recent years, MRTN is not as slow to adjust over time as it would be if each year had the same number of sites.

MRTN data

The MRTN data used for this research included 3,219 observations from on-farm yield response to nitrogen experiments conducted from 1999 to 2009 and 2013 to 2021 located across the north, central and southern regions of Illinois. The data were from four different projects.

The first project was from 1999 to 2008 at seven different sites. The second was from 2001 to 2004 and was done on farmer fields, in cooperation with the Illinois Department of Agriculture. The third project was composed of data from 2006 to 2008, with some additional trials from 2009 and was funded by a fertilizer tonnage fee administered by the Illinois Fertilizer Research and Education Council. The final project was from 2013 to 2021 and funded by a fertilizer tonnage fee administered by the Illinois Fertilizer Research and Education Council. All projects include corn-soybean rotations. Experiments from 70 Illinois counties out of 102 in the state are in the dataset. Nitrogen rates varied across locations, and six different rates per location were applied. Nitrogen rates ranged from 0 to 382 kilograms per hectare with an average of 147 kilograms per hectare. Typically, 0, 56, 112, 168, 224, and 280 kilograms of nitrogen were applied, and any additional nitrogen applied by the producer was added to these amounts. The yield data are treatment means for each site year, and all yield values were collected with a combine yield monitor, the weigh wagon method, or a small plot combine.

Production functions

Both stochastic linear response plateau (LRP) and stochastic quadratic plateau (QRP) models were estimated using the MRTN data. The SAS procedure PROC MCMC was used to estimate the model with Bayesian methods and weakly informative priors. The stochastic LRP model with time varying parameters is

$$(1) \qquad Y_{itj} = \min[(\beta_0 + \alpha_0 t) + (\beta_1 + \alpha_1 t)N_{itj}, P_0 + \alpha_2 t + u_{it}] + v_t + \gamma_{it} + \varepsilon_{itj}$$

where Y_{itj} is corn yield for the ith location for year t, j treatment, N_{itj} is the nitrogen level, β_0, α_0, β_1, α_1, P_0, and α_2 are parameters to be estimated, and $u_{it} \sim N_i(0, \sigma_u^2), v_t \sim N_i(0, \sigma_v^2), \gamma_{it} \sim N(0, \sigma_\gamma^2), \varepsilon_{itj} \sim N_i(0, \sigma_\varepsilon^2)$ with all four error terms being independent. The t is defined as $t = year - 2010$. The estimation procedure used 5,000 observations as burn in, a thinning rate of 20, and 20,000 simulated draws to generate each parameters' posterior distribution.

The priors used with the MRTN data are weakly informative. The priors for the mean parameters are normal distributions with large variances, so they have little influence on the posterior estimates. An improper inverse gamma prior was used for variances.

A stochastic quadratic model with time varying parameters following Cho et al. (2023) was considered, but not included here since the the Deviance Information Criterion (DIC) (Spiegelhalter et al., 2002) preferred the stochastic linear plateau.

Profit maximization

The optimal level of nitrogen is determined by maximizing expected profit. To perform the optimization, the SAS program PROC NLP is used. Expected profit is

$$(2) \qquad \max_{N_t \geq 0} \int E\pi(N_t|\theta)p(\theta|t)\, d\theta$$

where $E\pi(N_t|\theta)$ is expected profit given nitrogen and subject to the vectors of relevant parameters of the stochastic plateau, $E\pi = (p \cdot Y_t) - (r \cdot N_t)$ where p is the corn price, Y_i is the corn yield from the stochastic plateau model, r is the nitrogen price, and N_t is the amount of nitrogen applied from the stochastic plateau model, $\theta = (B_0, B_1, P_0, \sigma_u^2)$ is the vector of relevant parameters, and $p(\theta|t)$ is the posterior distribution for θ given year t. The calculation follows Brorsen (2013).

For the economic analysis, U.S. prices were used for calculating nitrogen and corn prices. Urea prices were used to determine a nitrogen price of $1.75 /kg of N. A corn price of $0.26/kg was used to determine the optimal nitrogen rates. These prices reflect January 2023 market prices for urea fertilizer (Quinn, 2023) in Omaha, Nebraska, and U.S. corn (USDA-NASS, 2023).

Data Intensive Farm Management

The second stage uses the Bayesian posteriors from the first stage as priors for site-specific functions estimated with on-farm experimental data. The on-farm experimental data from DIFM consist of 3,836 observations on a single field in north central Ohio over three years. The standard DIFM data cleaning procedure was used, which is described in Edge et al. (2024). All collected data were from corn following soybeans. This field was selected because it was the only DIFM field at the time that had three years of data and permission to let others use the data. The experimental design was a completely randomized design where treatments were assigned completely at random so that each experimental unit has the same chance of receiving any one treatment. The experiment is detailed in Table 1. We now know that a random design is not optimal and that alternative designs could have provided greater information (Poursina et al. 2023). The yield average for 2019 is lower than the averages of 2017 and 2021. The late planting date for 2019 could have contributed to the decrease in yield.

Table 1. On-Farm Experimental Data Selections from Single Ohio Field

	2017	2019	2021
Observations	141	101	91
Yield range (kg. /ha)	11,931-16,646	8,634-12,636	9,639-16,049
Yield average (kg. /ha)	14,567	11,272	13,068
Nitrogen range (kg. /ha)	185-246	176-282	182-320
Nitrogen average (kg. /ha)	218	230	262
Seeding range (thousand seed/ha)	74-95	73-96	59-104
Seeding average (thousand seed /ha)	85	87	81

A portion of the field was selected since estimation with the entire field was not computationally feasible. Using fewer observations reduces the computational time (even with the reduced dataset, the estimation still took over a week for 2021 data). Table 1 shows the means from the selected data.

The area of the field was selected[1] to have roughly the same number of observations across years. Due to missing values and different size grids, the number of observations varied by year. There were 141 observations selected from 1,373 observations from the 2017 data. In 2019, 101 observations were selected from 1,407 observations. In 2021, 91 observations were selected from 1,056 observations. GWR would have been impractical because it cannot handle missing values and has no ready way to deal with coordinates changing across years.

Spatially varying plateau model

Using the selected data, a spatially varying plateau model was estimated for each year:

(3) $$Y_i = \min\left[\beta_0 + \beta_1 N_i + \beta_2 S_i + \beta_3 S_i^2 + \beta_4 S_i N_i, P_i\right] + \varepsilon_i$$

where Y_i is corn yield for the ith location, N_i and S_i are the nitrogen and seeding rate for the ith location respectively. β_0, β_1, β_2, β_3, and β_4 are the corresponding coefficients for intercept, nitrogen, seeding rate, seeding rate squared, and the interaction between nitrogen and seeding rate. The posterior of the stochastic LRP model provided priors for β_0, β_1, β_4, and P_i of the spatially varying coefficients parameters.

(4) $$\boldsymbol{P} \sim MVN(\bar{p}\mathbf{1}, \boldsymbol{\Sigma})$$

where $\boldsymbol{P} = (P_1, P_2, \dots P_n)$ and $\boldsymbol{\Sigma} = cov\big(P(s), P(s')\big) = \sigma_p^2 \exp\left(-\frac{d_{s,s'}}{\rho}\right)$

where s and s' are two distinct locations in the field, $P(s)$ is the plateau for location s, σ_p^2 is the variance of the plateau, and $d_{s,s'}$ is the distance between s and s'. Bayesian methods are used to fit the model given in equation (3). The Hamiltonian Monte Carlo algorithm, which is faster and has a better convergence rate (Carpenter et al., 2017) than Metropolis-Hastings, is used through Stan to obtain posterior estimates. Four chains were used with 2,000 draws as warmup and 5,000 for estimation. The estimation took around 14 hours for each year (for 2021, one chain was slow to converge, and it took over a week). Convergence of the Markov Chain was checked using the Gelman-Rubin statistic (Gelman & Rubin, 1992).

The conditional autoregressive (CAR) and simultaneously autoregressive (SAR) models can provide faster computations (Poursina, 2022) than the exponential, but CAR and SAR are not applicable to data where the locations of the grids change every year.

The posterior from Equation 1 is used as a prior to forecast parameter distribution for 2021 using 2017, 2019, and 2021 data. The posterior predictive distribution is used to forecast parameters for 2021 locations from 2017, 2019, and 2021 data. New coordinates from 2021 are added to forecast the yield value.

Results and Discussion

Posterior distribution using the MRTN data

The mean and standard deviation of the posterior distribution for the parameters of the stochastic LRP model are presented in Table 2. The optimal nitrogen rate increased

[1] The model was estimated with a different portion of the field and estimates estimated spatial effects were also quite small.

21% from 2010 to 2021. The Illinois MRTN (Nafziger et al. 2022, Figure 4) has also shown increases in optimal N as new data were added and old data were dropped.

Table 2. Stochastic Linear Plateau Estimates on Illinois Corn Yields (kg /ha), 1999-2021

Parameter	Mean	Standard Deviation
Intercept	6614.5	138.9
Intercept time trend	0.1	17.9
Nitrogen	40.7	1.2
Slope time trend	0.5	0.2
Plateau	11853.4	142.5
Plateau time trend	204.0	20.2
Plateau variance	1728766.0	226626.0
Year random effect	4249889.0	345320.0
Error variance	1393012.0	44255.6
Optimal N 2010 (kg /ha)	160.2	
Optimal N 2021 (kg /ha)	193.3	
Deviance Information Criterion	48083.3	

Note: The selected price of nitrogen is $1.75/ kg., the price of corn is $0.26/kg. The data are 2799 observations from the Illinois Maximum Return to Nitrogen (MRTN) dataset.

Utilizing the time trend variable of the stochastic LRP model, the year 2023 optimal nitrogen rate was estimated using January 2023 prices for nitrogen, $1.75/kg, and corn, $0.26/kg (Quinn, 2023; USDA-NASS, 2023). The optimal nitrogen rate was 199 kg of nitrogen /ha. The nitrogen rate calculator (Nafziger, 2023) computed optimal nitrogen rates using the same prices for the three Illinois regions: North, Central, and South. The calculator estimated an optimal nitrogen rate of 187 kg /ha for the North region, 195 kg /ha for the Central region, and 218 kg /ha for the South region.

Production function estimates

Fig. 1 plots the stochastic LRP model using posterior means for 2010 and 2021. The intercept, slope, and plateau all increased over time. The intercept increased by 0.02 percent and the slope increased by roughly 13 percent. The plateau increased the most by roughly 28 percent, and thus the optimal nitrogen rate also increased.

Assefa et al., (2017) studied corn yield data from 1987 to 2015 and concluded that corn yields across the United States increased between 97 and 147 kg/ha/year. There are many reasons as to why corn yields have increased over time. Genetic improvements (Russell, 1991), increased plant densities (Assefa et al., 2018), and earlier planting dates (Tannura et al., 2008) have all been suggested as explanations for increasing corn yields. All estimated variances were quadrupled when used as priors to allow the information from the field to have more impact on the posterior distribution.

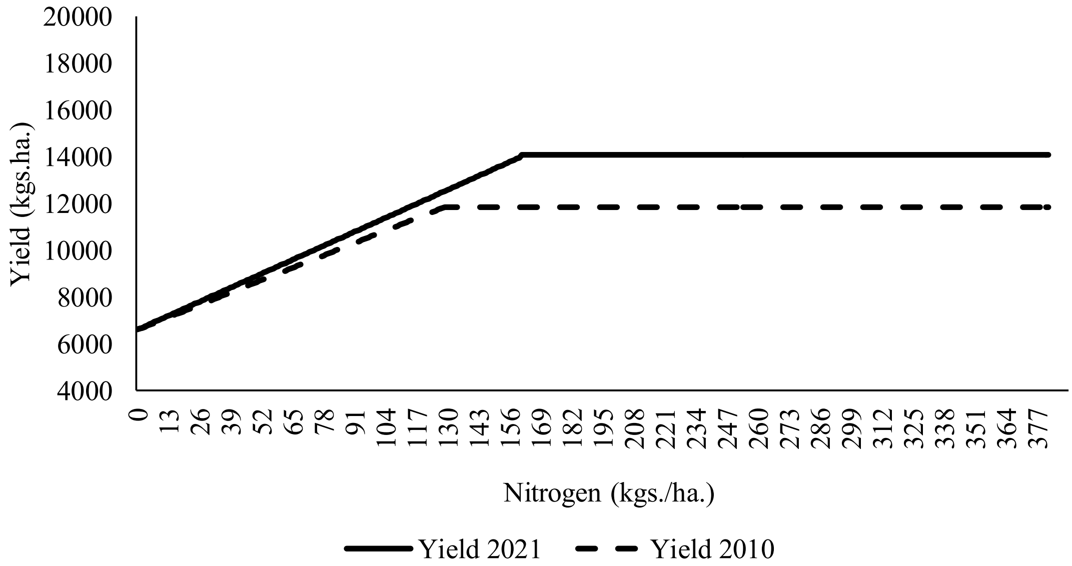

Fig. 1. Expected Stochastic Linear Plateau Models for 2010 and 2021

DIFM data

Treatment means for yield for the three years of on-farm experiment are in Fig. 2. High seeding rates were between 89,000 and 104,000 seeds/ha, medium between 74,000 and 88,999 seeds/ha, and low less than or equal to 73,999 seeds/ha. The figure highlights the difficulties in using this data. High seeding rates appear beneficial in 2021 but had no effect in other years. The curves are relatively flat for nitrogen, except for the medium seeding rate in 2019 and the high seeding rate in 2021 where the plateau was not reached with the highest levels of nitrogen applied.

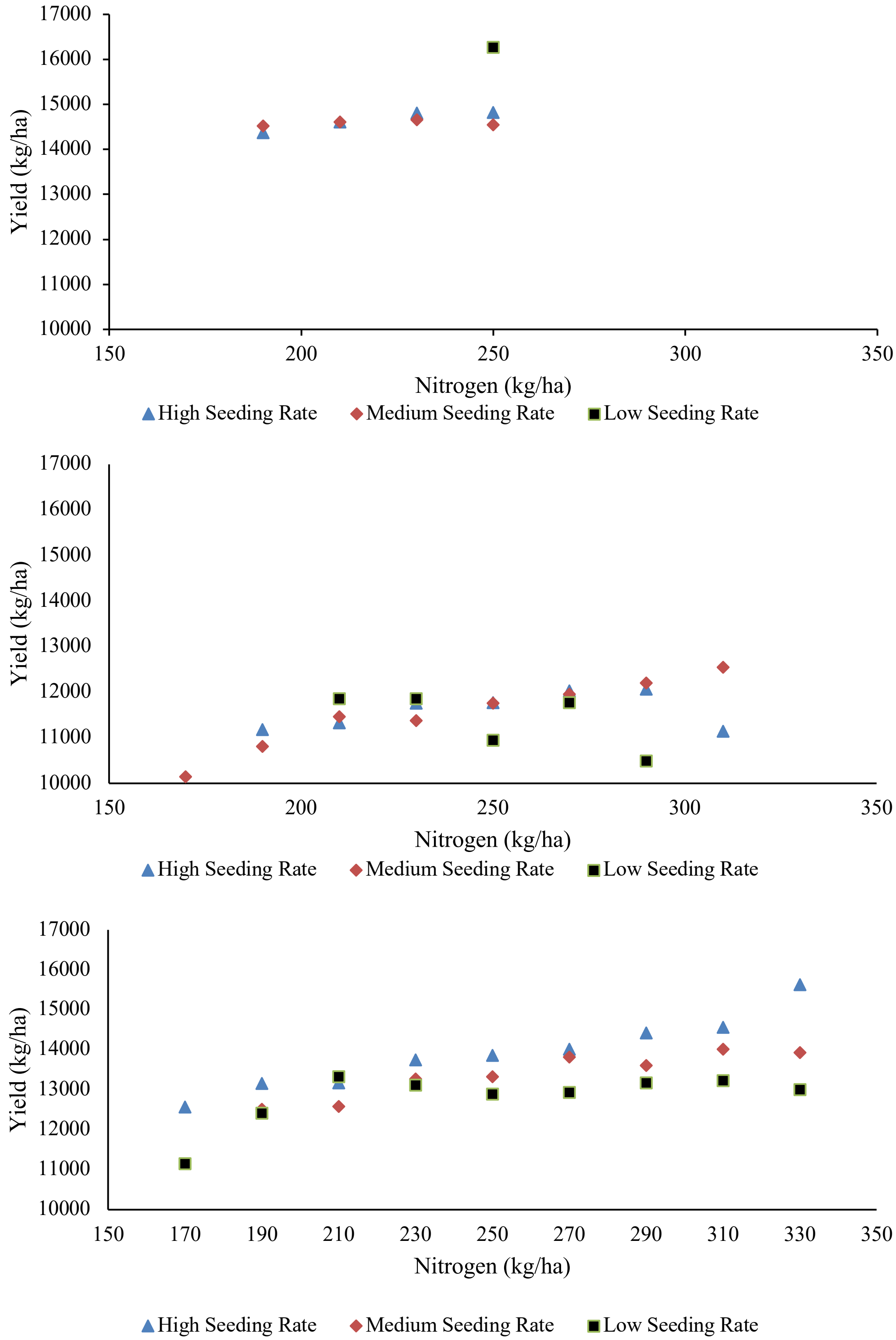

Fig. 2. Mean Yield Reponses to Nitrogen and Seeding Rate for 2017 (top), 2019 (middle), and 2021 (bottom)

Spatially Varying Plateau Estimates
The stochastic LRP model is used as a prior when estimating the spatially varying plateau model. The estimated spatially varying coefficient models are used to predict corn yield response for 2021 selected locations and using response data from 2017, 2019, and 2021.

The model produces a different spatially varying value of the plateau for each location. The spatially varying estimates are quite small. The plateau's latent spatial process of 2021 locations from 2017, 2019, and 2021 data were used to create Fig. 3. The maps show no distinct pattern across years and the plateau varies little across the field, which foretells that precision nitrogen application is of little value in this field.

Patterson (2023) conducted multiple tests on the out-of-sample predictions and found no predictive accuracy. Many past researchers have found uniform nitrogen rates are more profitable than variable rate applications (Isik & Khanna, 2002; Thrikawala et al., 1999; Edge 2022). Variable rate applications have been profitable given sufficient spatial variability (Roberts et al., 2000). The area of the field studied here did not have sufficient spatial variability to derive a benefit from using variable rate nitrogen application.

Optimal nitrogen and seeding rate
The optimal nitrogen and seeding rates were calculated for each prediction year using the prediction parameter means from the output of the spatially varying coefficient model estimation. A seed corn price of $2.00 per thousand seeds (Lauer & Stanger, 2023), corn price of $0.26 per kg (Quinn, 2023), and nitrogen price of $1.75 per kg (USDA-NASS, 2023) were used. Optimal levels were constrained to be within the range of the data. All optimal levels were corner solutions. What is shown is that the seeding rates and nitrogen levels used were so high that almost all observations were on the plateau. To learn more about optimal levels, lower rates of nitrogen and lower seeding rates would need to be considered. This conclusion is consistent with Poursina and Brorsen (2024) who find that an optimal on-farm experiment should include a small number of plots with a low level of input.

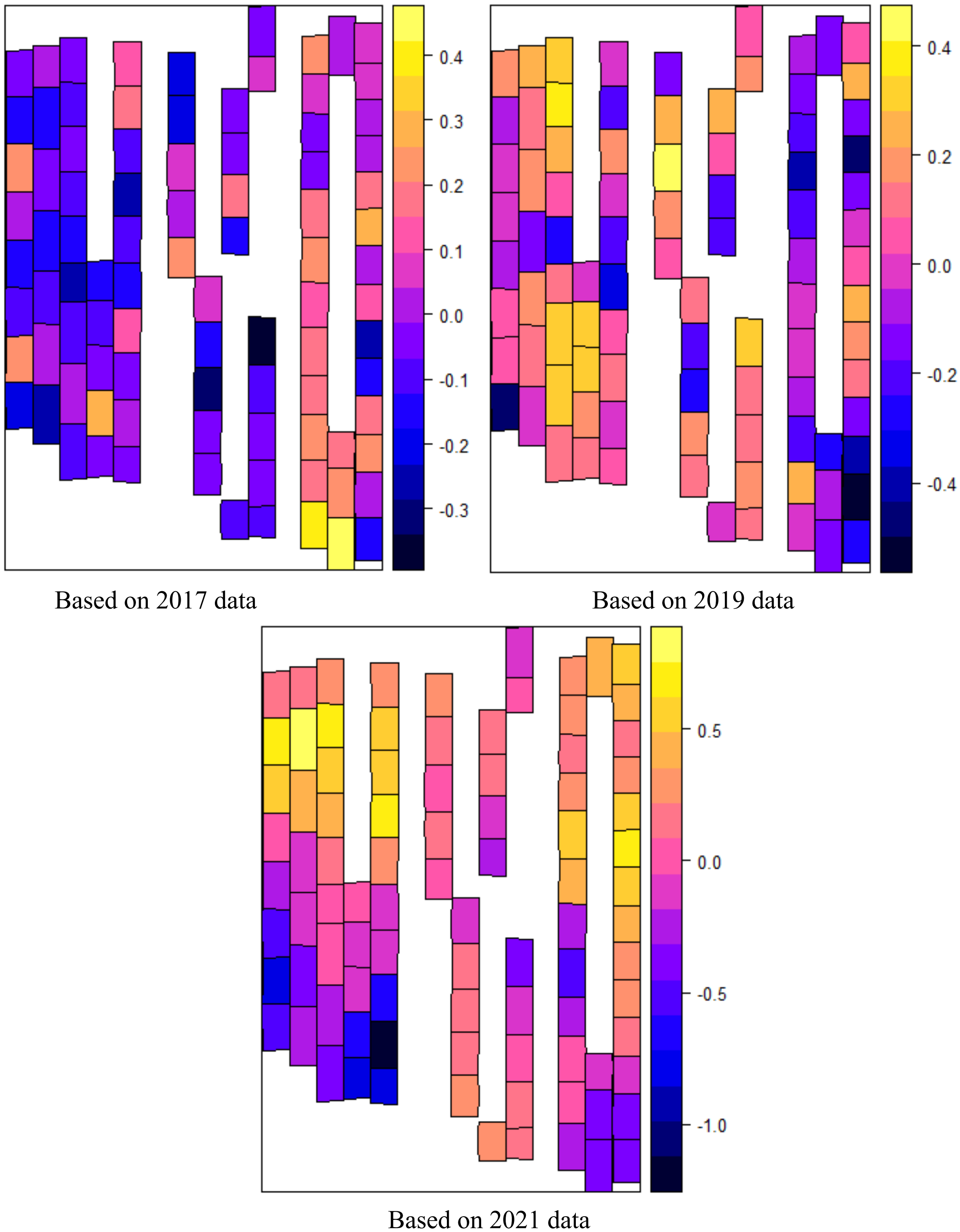

Fig. 3. Predicted Gaussian spatial process values for 2021 based on the data from 2017, 2019, and 2021

Conclusions

The first goal was to develop informative Bayesian priors. The goal was achieved by estimating a stochastic linear plateau model using data from the MRTN database and incorporating a time trend that accounts for increasing corn yields over time. The intercept, slope, and plateau all increased over time. The plateau increased the most by 28 percent, and thus the optimal nitrogen rate also increased. The optimal nitrogen rate from 2010 to 2021 increased by almost 21 percent. The estimated posterior distribution of the stochastic linear plateau is used as a prior for completing the second goal of this research which is to estimate a spatially varying coefficient model to determine accurate site-specific nitrogen rates. After estimating the spatially varying coefficient model, the estimated plateaus revealed little spatial variability across the field which limited the benefit of applying variable rate nitrogen. Poursina (2022) suggests experimenting on only a part of the field, which would increase the importance and dependence upon the priors. The general approach used here might prove useful for determining uniform rate recommendations even if it did not aid in variable rate recommendations. This research is consistent with previous research that found variable rate nitrogen may not be profitable when the only information available is the location of the plot. Only a section of the field was analyzed due to limits on the computational time, which is a limitation that will need to be overcome if this approach is to be commercially viable.

The uniformity of the field studied likely contributed to variable rate nitrogen not being profitable. Another limitation is the need for lower seeding rates (or using a constant seeding rate) and lower nitrogen rates in order to get more precise estimates. Variable rate nitrogen applications can still be profitable in a field with higher spatial variability. Finally combining the approach used here with other sources of data such as remote sensing might lead to profitable variable rate nitrogen application.

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