

RESEARCH ON STRAIGHT-LINE PATH TRACKING CONTROL METHODS IN AN AGRICULTURAL VEHICLE NAVIGATION SYSTEM

Taochang Li¹, Jingtao Hu², Lei Gao³, Hechun Hu⁴, Xiaoping Bai⁵, Xiaoguang Liu⁶

*1, 2, 3, 4, 5, 6: Department of Information Service and Intelligent Control
Shenyang Institute of Automation, Chinese Academy of Sciences
Shenyang, China*

*1, 2, 3, 5, 6: Graduate School
Chinese Academy of Sciences
Beijing, China*

ABSTRACT

In the precision agriculture (PA), an agricultural vehicle navigation system is essential and precision of the vehicle path tracking is of great importance in such a system. As straight line operation is the main way of agricultural vehicles on large fields, this paper focuses on the discussion of straight-line path tracking control methods and proposes an agricultural vehicle path tracking algorithm based on the optimal control theory. First, the paper deduces a relative kinematics model of agricultural vehicles based on lateral deviation and heading errors between vehicles and paths. And then a linear quadric (LQ) optimal controller is introduced to improve the control precision and other performance index, such as stability and fast response. The stability of the controller at different speeds is also discussed and the stability condition according to Lyapunov stability theory is proved. Finally, the feasibility of the control algorithm is verified by a series of experiments with a combine operating on a road. The results show that the algorithm proposed in the paper yields satisfactory effects on the straight-line path tracking of agricultural vehicles. At slow speeds, the range of lateral position deviation of the straight-line path tracking is approximately from -0.08m to 0.12m and the mean value of the lateral position deviation is 0.05m.

Keywords: Precision agriculture, Navigation, Agricultural vehicles, Kinematics model, Linear quadric optimal control

INTRODUCTION

In the context of precision agriculture, automatic navigation for agricultural vehicles is one of the key technologies to realize precision farming operations, such as planting, fertilization, spraying, tillage, cultivation, etc. Research on agricultural vehicles navigation has become very popular in the last ten years and farmers will be using affordable, dependable autonomous vehicles for agricultural applications in the near future.

For most of the farming operations mentioned above, the path tracking accuracy of an agricultural vehicle navigation system is essential. From the perspective of control, there is a long history in dealing with the path tracking control of vehicles. Generally there are mainly two types of control methods for path tracking, the kinematics model-based and the dynamics model-based method. Huang et al. (Huang et al., 2010) used the BP neural network to determine look-ahead distance for a pure pursuit model and then a desired steering angle was obtained based on the pure pursuit model and a simplified bicycle kinematics model. Luo et al. (Luo et al., 2009) developed a navigation control system for Dongfanghong X-804 tractors and the navigation controller was developed based on Ellis kinematics model. Ding and Wang (Ding and Wang, 2010) constructed a fuzzy PD controller based on a simplified two-wheel vehicle model in a vision navigation system. Zhu et al. (Zhu et al., 2007) created a suboptimal reference course and designed a path-tracking controller based on a vehicle kinematic model for headland turning of a tractor. Since the kinematics model-based control method didn't consider the effect of the dynamics parameters, some researchers developed the dynamics model-based method.

Eaton et al. (Eaton et al., 2009) investigated a back-stepping controller taking the effects of steering dynamics into account. The controller compensated directly the realistic steering dynamics by a back-stepping controller rather than a low level steering controller. Zhang and Qiu (Qiu, 2002; Zhang and Qiu, 2004) developed a dynamic path search algorithm for tractor automatic navigation and used on-board RTK-DGPS (Real time kinematic differential GPS) and FOG (Fiber optic gyroscope) sensors to provide a real-time tractor posture measurement. Derrick et al. (Derrick and Bevly, 2008; Derrick et al., 2008; Derrick and Bevly, 2009) proposed a model reference adaptive control method based on a yaw dynamics model to compensate yaw rate variations due to the changes of implements attached to the tractor.

In a word, to answer the growing high precision demand in PA, many control methods have been proposed and satisfactory results have been reported. However, these methods have some strict application constrains, for example, dynamic model parameters are hard to obtain, controllers are difficult to implement, the controller's design requires empirical knowledge and performance index is not optimum. In view of the above problem, in this paper we use the deduced relative kinematics model of agricultural vehicles and propose a new navigation controller for agricultural vehicles based on the linear quadric optimal control method.

The remainder of this paper is divided into four sections. First, the kinematics model of the agricultural vehicle is introduced and a relative

kinematics model is deduced based on lateral deviation and heading errors between vehicles and paths in section 2. Then a linear quadric optimal controller is presented based on the deduced relative kinematic model and the controller's stability at different speeds is proved by Lyapunov stability theory in section 3. Finally, the efficiency of the method is validated by experiment in section 4, and conclusions are drawn in section 5.

RELATIVE KINEMATICS MODEL

A kinematics model is applied to describe the agricultural vehicle motion. Fig. 1 illustrates a bicycle kinematics model and the relative relation between vehicles and paths.

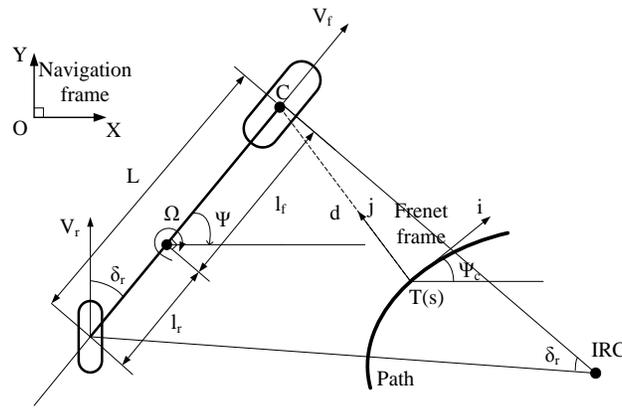


Fig. 1. The relative relation between the vehicle and the path

As shown in Fig. 1, we define the navigation frame and the Frenet frame. Suppose the vehicle mass is carried on the front axle totally, we can choose point C as the control point of the vehicle. The point C projects orthogonally to the point T(s) on the path and is characterized by the coordinates (X, Y) in the navigation frame, equivalently by the coordinates (0, d) in the Frenet frame. Then, we can get the agricultural vehicle kinematics model in the navigation frame as follows:

$$\begin{cases} \dot{X} = V_f \cos(\psi) \\ \dot{Y} = V_f \sin(\psi) \\ \dot{\psi} = V_f \tan(\delta_r) / L \end{cases} \quad (1)$$

Where

X is the lateral coordinate of the vehicle in the navigation frame.

Y is the longitudinal coordinate of the vehicle in the navigation frame.

V_f is the longitudinal speed of the vehicle.

ψ is the orientation of the vehicle centerline with respect to the X axis of the navigation frame.

δ_r is the steering angle of a rear wheel.

L is the wheelbase of the vehicle.

At first, we deduce the relative kinematics model whose state variables are

lateral deviation and heading deviation in order to transform the tacking control problem into a stabilization control problem. We use the following notations:

d is the lateral deviation of the agricultural vehicle with respect to a reference path. When the vehicle locates on the left side of the path, the value of d is negative, otherwise it is positive.

s is the curvilinear coordinate of point T(s) along the reference path.

$c(s)$ is the curvature of the reference path.

ψ_c denotes the tangent orientation at point T(s) on the reference path in the navigation frame.

θ_e stands for the heading angle deviation of the vehicle with respect to the reference path.

We define N-derivative as the time derivative of a vector \vec{r} in the navigation frame and F-derivative in the Frenet frame as follows:

$$\text{N-derivative} := \frac{N d}{dt} \left(\begin{matrix} N \\ \vec{r} \end{matrix} \right) = \dot{X}I + \dot{Y}J + \dot{Z}K \quad (2)$$

$$\text{F-derivative} := \frac{F d}{dt} \left(\begin{matrix} F \\ \vec{r} \end{matrix} \right) = \dot{x}i + \dot{y}j + \dot{z}k \quad (3)$$

Consequently, we can prove the following relation easily.

$$\frac{N d}{dt} \left(\begin{matrix} F \\ \vec{r} \end{matrix} \right) = \frac{F d}{dt} \left(\begin{matrix} F \\ \vec{r} \end{matrix} \right) + \omega_F \times \begin{matrix} F \\ \vec{r} \end{matrix} \quad (4)$$

Let $\begin{matrix} F \\ \vec{r} \end{matrix} = \begin{matrix} F \\ \overline{TC} \end{matrix}$, then we can deduce the relation directly as follows:

$$\frac{N d}{dt} \left(\begin{matrix} F \\ \overline{TC} \end{matrix} \right) = \frac{F d}{dt} \left(\begin{matrix} F \\ \overline{TC} \end{matrix} \right) + \omega_F \times \left(\begin{matrix} F \\ \overline{TC} \end{matrix} \right) \quad (5)$$

$$\text{Where } \begin{matrix} F \\ \overline{TC} \end{matrix} = \begin{bmatrix} 0 \\ d \\ 0 \end{bmatrix} \text{ and } \omega_F = \begin{bmatrix} 0 \\ 0 \\ \dot{s}c(s) \end{bmatrix}$$

Therefore, we can deduce the following relation:

$$\begin{bmatrix} \dot{s} \\ \dot{d} \\ 0 \end{bmatrix} = \begin{bmatrix} \cos \psi_c & \sin \psi_c & 0 \\ -\sin \psi_c & \cos \psi_c & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{X} \\ \dot{Y} \\ 0 \end{bmatrix} + \begin{bmatrix} \dot{s}c(s) \\ 0 \\ 0 \end{bmatrix} \quad (6)$$

According to (1) and (6), we can deduce the following relative kinematics model (7) which indicates the relative position and attitude relations between the agricultural vehicle and the path.

$$\begin{cases} \dot{s} = \frac{V_f \cos \theta_e}{1 - dc(s)} \\ \dot{d} = V_f \sin \theta_e \\ \dot{\theta}_e = \frac{V_f \tan \delta_r}{L} - \frac{V_f c(s) \cos \theta_e}{1 - dc(s)} \end{cases} \quad (7)$$

For the sake of our control purpose in this paper, we choose straight-lines as the tracked-paths. However, the limitation does not lose generality. Let $c(s)$ be zero in equation (7), we get the following differential equation:

$$\begin{cases} \dot{d} = V_f \sin \theta_e \\ \dot{\theta}_e = \frac{V_f \tan \delta_r}{L} \end{cases} \quad (8)$$

We can employ first-order Taylor series to approximate equation (8) if both θ_e and δ_r are small. The small angle hypothesis is reasonable for agricultural vehicles tracking a straight-line path. Consequently we write the model by the state equation whose state variables are $[d \ \theta_e]^T$ as follows:

$$\dot{x} = Ax + B\delta_r \quad (9)$$

$$\text{Where } x = [d, \theta_e]^T, \quad A = \begin{bmatrix} 0 & V_f \\ 0 & 0 \end{bmatrix} \text{ and } B = \begin{bmatrix} 0 \\ \frac{V_f}{L} \end{bmatrix}.$$

CONTROL METHODS

LQ optimal control method

If a control system is linear, the performance index function is the integral of quadratic functions of state and control variables according to the optimal control theory. In this situation, the optimal control problem is known as a LQ optimal control problem. The control law solved by a LQ optimal control problem is linear function of state variables, so the closed-loop optimal control can be executed by the state feedback. In LQ optimal control problems, there are LQ regulators (LQR) and LQ trackers. Here we use LQR in our control tasks. LQR has two cases: finite time state regulator and infinite time state regulator. During the design process of a finite time state regulator, we need to solve differential Riccati equations and the designed controllers are difficult to execute. In view of engineering background, if we consider the steady states of the controlled problem only, the differential Riccati equation can be reduced to an algebraic Riccati equation. The solution matrix of Riccati equations will then tend to a constant matrix and the closed-loop optimal control can be executed easily. In this case, the LQ optimal control problem is referred to as an infinite time state regulator problem. According to the above discussion, the infinite time state regulator is used in this paper and it has two merits as follows:

1) If a system deviates from an equilibrium state due to disturbances, the system can return to the equilibrium state optimally and there are not steady state errors.

2) The closed-loop system is asymptotically stable and the optimal state feedback matrix is constant.

Control method based on LQR

In this paper, we propose a straight-line path tracking control method based on LQR for agricultural vehicles. Considering that the speed is relatively slow and stable when the agricultural vehicle is operating in the field, we can suppose that the speed of the agricultural vehicle is constant and the system demonstrated by

the state equation (9) is linear time-invariant. We will give the stability condition of the closed-loop control system when the speed varies in next section. Through the above discussion, we can use the infinite time state regulator to work out the agricultural vehicle navigation control law. The control diagram of the agricultural vehicle control system is shown in Fig. 2.

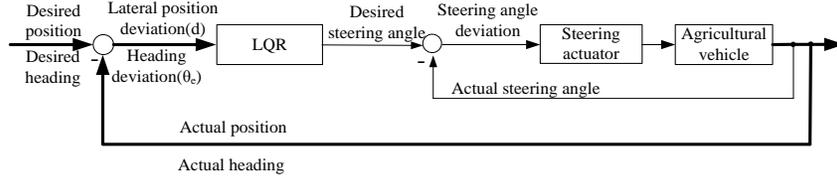


Fig. 2. The control diagram of the agricultural vehicle navigation system

According to equation (9) and the infinite time state regulator theory, we use the performance index function (10) and obtain the desired steering angle as (11).

$$J = \frac{1}{2} \int_0^{\infty} ([d \ \theta_e] Q \begin{bmatrix} d \\ \theta_e \end{bmatrix} + \delta_r^T R \delta_r) dt \quad (10)$$

Where $Q = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$, $R = r$ and $a > 0, b > 0, r > 0$.

$$\begin{aligned} \delta_r &= -R^{-1} B^T P [d \ \theta_e]^T \\ &= [-\frac{V_f}{rL} p_{12}, -\frac{V_f}{rL} p_{22}] [d \ \theta_e]^T \end{aligned} \quad (11)$$

Where p_{12}, p_{22} are the elements of symmetric positive-definite solution matrix $P = \begin{bmatrix} p_{11} & p_{12} \\ p_{12} & p_{22} \end{bmatrix}$ of the algebraic Riccati equation.

Solving the algebraic Riccati equation (12), we can obtain (13).

$$PA + A^T P - PBR^{-1}B^T P + Q = 0 \quad (12)$$

Where $A = \begin{bmatrix} 0 & V_f \\ 0 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 0 \\ V_f \\ L \end{bmatrix}$.

$$\begin{cases} p_{12} = \frac{L}{V_f} \sqrt{ar} \\ p_{22} = \frac{L}{V_f} \sqrt{br + 2Lr\sqrt{ar}} \end{cases} \quad (13)$$

According to (11) and (13), the desired steering angle can be described further as (14).

$$\begin{aligned} \delta_r &= -\left[\frac{\sqrt{ar}}{r}, \frac{\sqrt{br + 2Lr\sqrt{ar}}}{r} \right] [d \ \theta_e]^T \\ &= -K [d \ \theta_e]^T \end{aligned} \quad (14)$$

Where $K=[k_1, k_2]$ is the state feedback matrix with $k_1 = \frac{\sqrt{ar}}{r} > 0$ and $k_2 = \frac{\sqrt{br + 2Lr\sqrt{ar}}}{r} > 0$.

Stability at different speeds

In the previous section, we regard the speed of agricultural vehicles as constant. Generally speaking, we cannot guarantee that the vehicle speed is always unvarying in field. Therefore the system matrix A and the control matrix B will change at different speeds.

According to (9) and (14), the closed-loop system is (15).

$$\dot{x} = A_c x = \begin{bmatrix} 0 & V_f \\ \frac{-k_1 V_f}{L} & \frac{-k_2 V_f}{L} \end{bmatrix} x \quad (15)$$

With regard to the stability of the closed-loop system (15) at different speeds, we propose Theorem 1.

Theorem 1: For any positive speed V_f , the equilibrium point $x_e = [0, 0]^T$ of the closed-loop system (15) is asymptotically stable by using the control law given in (14) with any $k_1 > 0$ and $k_2 > 0$.

Proof:

Constructing a Lyapunov function candidate as follows:

$$V(x) = x^T M x$$

$$\text{Where } M = \begin{bmatrix} m_{11} & m_{12} \\ m_{12} & m_{22} \end{bmatrix} = \begin{bmatrix} \frac{k_1^2 + k_2^2 + k_1 L}{2k_1 k_2 V_f} & \frac{L}{2k_1 V_f} \\ \frac{L}{2k_1 V_f} & \frac{k_1 L + L^2}{2k_1 k_2 V_f} \end{bmatrix}.$$

Let

$$\Delta_1 = m_{11} = \frac{k_1^2 + k_2^2 + k_1 L}{2k_1 k_2 V_f}$$

$$\Delta_2 = \begin{vmatrix} m_{11} & m_{12} \\ m_{12} & m_{22} \end{vmatrix} = \frac{k_1 L [(k_1 + L)^2 + k_2^2]}{4k_1^2 k_2^2 V_f^2}$$

Since both Δ_1 and Δ_2 are positive for any $k_1 > 0$, $k_2 > 0$ and $V_f > 0$, then according to Sylvester's criterion, matrix M is positive definite and then the Lyapunov function $V(x)$ is positive definite.

According to (15), the derivative of $V(x)$ is given as follows:

$$\begin{aligned}
\dot{V}(x) &= x^T M\dot{x} + \dot{x}^T Mx \\
&= x^T (MA_c + A_c^T M)x \\
&= -x^T Ix
\end{aligned}$$

Where I is the identity matrix.

Since $\dot{V}(x)$ is negative when x is not equal to zero, the closed-loop system (15) is asymptotically stable according to Lyapunov stability theory. \square

According to **Theorem 1** the control system will be asymptotically stable no matter what the positive speed is in theory. However, the actuator response with respect to the desired steer angle input is delayed. Because of the delay limit, the speed of the agricultural vehicle should not be too fast. Fortunately, in most precision farming applications, the speed ranges from 0.5m/s to 2m/s and we can neglect the delay impact in the speed range.

EXPERIMENTAL RESULTS

Experimental platform

The developed control method has been tested and verified by a series of experiments on a combine as shown in Fig. 3. The geometric and inertial parameters of the combine are shown in Table 4-1.

Table 1. Experimental vehicle parameters

Parameter	Value
mass	9910kg
wheelbase	3750mm
turning radius	8000mm
front track width	2445mm
rear track width	2230mm



Fig. 3. The experiment combine

Description of the experiments

The experimental platform described above is equipped with a data acquisition system and the navigation system based on the control method proposed in this paper. Experiments of straight lines tracking are performed at different speeds. And the description of the experiment is as follows.

Experiment 1:

Step 1: Open GPS reference station system and the data acquisition system , then carry out magnetic field correction and magnetic declination compensation of a heading sensor.

Step 2: Set the AB path from east to west approximately.

Step 3: Start automatic navigation of the agricultural vehicle on the AB path.

Step 4: Repeat step 3 at the speed 0.8m/s and 1m/s.

Experiment 2:

Step 1~2 are the same as those in experiment 1.

Step 3: Start automatic navigation of the agricultural vehicle from a point about 1.3m away from the AB path.

Step 4: Repeat step 3 at the speed 0.8m/s and 1m/s.

In the experiments we choose $a = r = 1.5$, $b = 1$ and $L = 3.75\text{m}$. We may determine the state feedback matrix $K = [1.00 \quad 2.86]$ according to the control law (14) designed based on the infinite time state regulator theory. Actually, the K will change a little because of the existence of disturbance. The experiment results are shown in next section.

Path tracking results and discussions

Fig. 4 shows the agricultural vehicle's track following path AB and Fig. 5 indicates the path tracking errors in experiment 1. The range of the lateral position deviation is about from -0.08m to 0.12m . Mean value of the lateral position deviation is 0.05m . Variance of the lateral position deviation is 0.0033m .

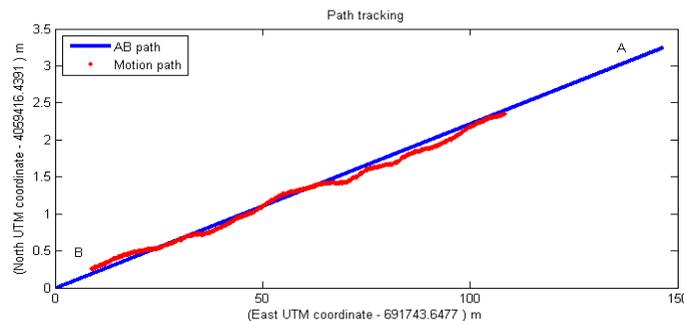


Fig. 4. Agricultural vehicle's track following path AB in experiment 1

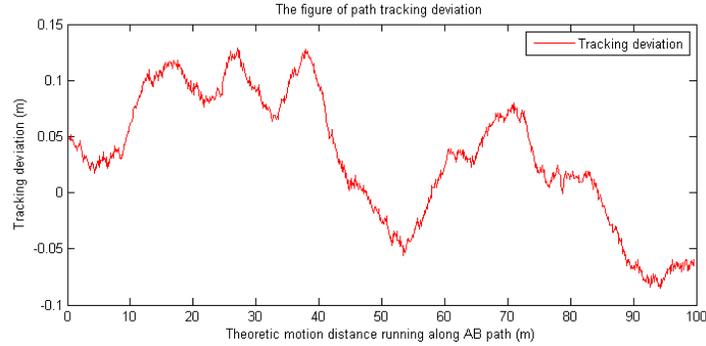


Fig. 5. The lateral position deviation in experiment 1

Fig. 6 demonstrates that the agricultural vehicle starting from a point 1.3m away from the AB path approaches the path gradually and tracks the path finally. Fig. 7 explains the variation of the lateral position deviation when the vehicle is tracking the path gradually. The agricultural vehicle tracks the path after running along the path AB around 10m which is about two times the length of the vehicle equivalently. Fig. 8 shows the path tracking errors after tracking the AB path in experiment 2. The range of the lateral position deviation is about from -0.08m to 0.04m. Mean value of the lateral position deviation is 0.04m. Variance of the lateral position deviation is 0.0009m.

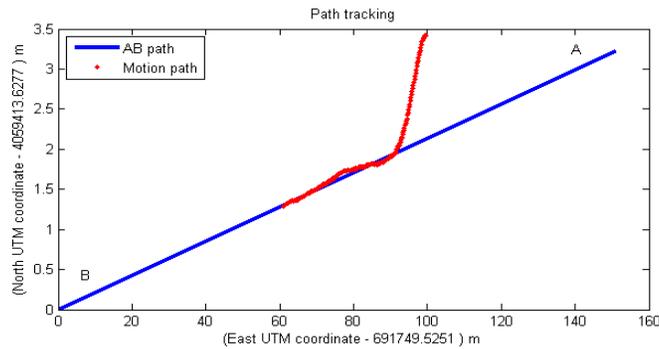


Fig. 6. Agricultural vehicle's track following path AB in experiment 2

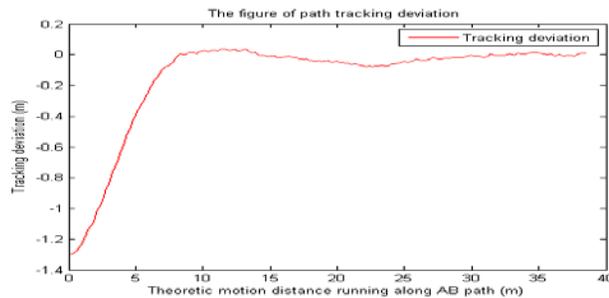


Fig. 7. The lateral position deviation in experiment 2

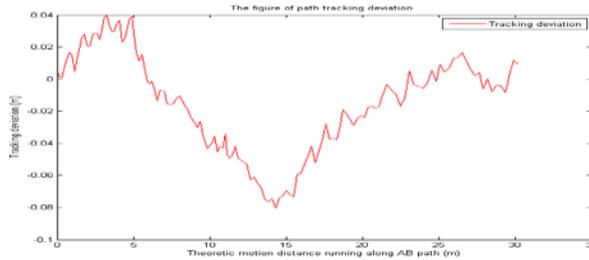


Fig. 8. The lateral position error after tracking the path in experiment 2

Through the experiment results, we can conclude that the max value of the lateral position deviation is less than $\pm 12\text{cm}$ and the control system has very good response speed. Therefore, the feasibility of the control algorithm is verified and the algorithm proposed in the paper yields satisfactory effects on the straight-line path tracking of agricultural vehicles.

CONCLUSIONS AND FUTURE WORK

In this paper, a navigation control method of straight-line path tracking based on LQR is presented. First, the paper deduces a relative kinematics model of agricultural vehicles based on lateral deviation and heading errors between vehicles and paths. And then we develop an infinite time state regulator to control the agricultural vehicle to track straight-line paths and also prove the stability of the closed-loop control system at different speeds. Finally, In order to test and verify the proposed method, we design two kinds of experiments. Experiment results show that the method can meet the requirement of agricultural vehicles in farming operations.

As the agricultural vehicles inevitably suffer from sliding due to changes of soil conditions, running at high speeds, or tracking a curve, some improvements can be expected. We need to design a robust or adaptive control method to eliminate them. And we may design a nonlinear control method to deal with the curve tracking problem.

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