A Comparative Study of Field-Wide Estimation of Soil Moisture Using Compressive Sensing

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Abstract. *In precision agriculture, monitoring of soil moisture plays an essential role in correct decision making. In practice, regular mesh installation, or large random deployment of moisture sensors over a large field is not possible due to cost and maintenance prohibitions. Consequently, direct measurement of moisture is possible at only a few points in the field. A value for the moisture may then be estimated for the remaining areas using a variety of algorithms.*

It is shown that although soil moisture varies spatially, the values are typically spatially correlated. Consequently, they have a sparse representation in the frequency domain. For such signals, compressive sensing (CS) has proven to be an effective tool in estimating the missing variable values, from sensed values.

CS theory is based on a l_0-norm optimization problem which is non-deterministic polynomial-time (NP) hard problem and requires an exhaustive search over all possible locations of the nonzero entries in the corresponding sparse signal. For most real-life applications, this optimization translates into a very large-scale problem which takes substantial time and computing resources to solve. This is usually circumvented by instead using an approximation of the l_0-norm.

The l_0-norm presents two challenges when incorporated into an optimization problem. It is both non-smooth and non-linear. Smooth approximations of the zero norm exist in various linear and non-linear forms. The nature of each approximation makes it more apt for a different type of application (with respect to size of the problem, nonconvexity of the original problem, and the requisite computational speed). In this paper, some different approximations of the zero norm are compared to determine which type is more suited to soil moisture application problems.

The data set that is used for numerical experiments is described. It is extracted from the simulation of a simple field using the state-of-the-art TIN-based Real-time Integrate Basin Simulator (tRIBS). The problem is then solved for different approximations of l_0-norm and a detailed comparative study is presented.

Keywords. *Moisture monitoring, Precision agriculture, Compressive sensing, l_0-norm optimization.*

Introduction

The Increasing population and demand on food sources on the one hand, and global water crisis on the other hand, create a strong motivation for governments, farmers, engineers, scientists and researchers to develop more efficient irrigation methods (Rogers, Llamas, & Martinez-Cortina, 2006). In precision agriculture we seek to cut wasteful use of water by delivering the correct amount to the crop over its growth cycle and in response to varied environmental conditions (Zhang, 2015). This will improve yield, leading to an overall water saving.

However, precise closed-loop control of irrigation, and analysis of moisture on a field is not possible without gathering sufficient and accurate information. Traditional moisture measurements method as those found in small controlled operations such as green houses (Hamouda & Elhabil, 2017) are not scalable to large operations due to cost and maintenance issues. Remote sensing methods are scalable to such large fields (Gruhier et al., 2010), however in-demand data at the correct resolution are expensive, or not available.

In other engineering applications with similar challenges, a successful approach has been to combine direct measurements with estimation theorems such as compressive sensing (CS). CS theory is based on a l_0-norm optimization problem which is NP-hard problem and requires an exhaustive search of all possible locations of the nonzero entries. The huge computational burden means that for large scale and real-time problems, it is nearly impossible to solve directly. The l_1 minimization yields similar (or even the same) result as the l_0 minimization in many cases of practical interest (Patel & Chellappa, 2013). However, in some problems it is in fact an extremely poor approximation to use (Candes, Wakin, & Boyd, 2008).

Compressive Sensing has been applied to the moisture estimation problem, but in a limited capacity (X. Wu, Wu, Liu, & Zheng, 2011). Effective application of CS theory to the moisture problem would imply that direct sensor placement will be limited to only a few points in the field, and the recorded data shall be used to estimate the moisture value at all other field locations. The scarcity of CS theory application in the moisture estimation problem means that a good insight on the type of approximation which leads to least error in the moisture estimation problem, is lacking. Accordingly. the main purpose of this paper is to present a comparative study between several different methods for approximation of the l_0-norm optimization when applied to the moisture estimation problem. We conclude the paper by giving a recommendation based on our findings.

The remainder of this paper is organized as follows. At first, CS theory is explained briefly. Next, CS theory is formulated and applied to the moisture estimation problem. Subsequently, a section is dedicated to the discussion about data set that is utilized for numerical experiments. The optimization problem is then solved with simple l_1-norm approximation and some more sophisticated approximations and the results are compared.

Compressive Sensing (CS) Theory

CS (Donoho, 2006) is a concept in information theory and signal processing that is useful for reconstructing sparse signals from measurements at rates below the Nyquist rate (Patel & Chellappa, 2013).

Let **x** be a discrete time signal which can be considered as a N×1 column vector in $\mathbb{R}^N$. **x** is K-sparse if it has only K nonzero elements. A signal is considered as a sparse signal if K<<N.

The l_p – norm of a vector is defined as,

$$\|x\|_p = \left(\textstyle\sum_i |x_i|^p\right)^{\frac{1}{p}}, \tag{1}$$

and the l_0-norm is defined as the limit p→0 of the l_p-norm,

$$\|x\|_0 = \lim_{p\to 0} \|x\|_p^p = \lim_{p\to 0} \textstyle\sum_i |x_i|^p. \tag{2}$$

The l_0-norm of a signal counts the number of nonzero elements in the signal. Thus, if **x** is K-

sparse, $\|x\|_0 = K$.

In practice many real signals are not exactly sparse, instead they are compressible. A signal is compressible if the magnitude of the coefficients (when sorted in a decreasing order) decays according to a power law (Patel & Chellappa, 2013). CS theory can be applied to compressible signals as well. In this paper, the phrase "sparse signal" is used to refer to both exactly sparse, as well as compressible signals.

Let **x** be a sparse signal of size *N*. It is possible to reconstruct it from *M* samples where $y_{M\times 1} = \Phi x$. $\boldsymbol{\Phi}$ is usually referred to as the measurement matrix in CS theory. The problem of finding the sparsest solution can be formulated as the following optimization problem (Patel & Chellappa, 2013),

$$\hat{x} = \arg\min_{x} \|x\|_0 \text{ subject to } \ y = \Phi x. \quad (3)$$

where $\boldsymbol{\Phi}$ and *M* must satisfy sets of constraints, some of which shall be discussed in the next section (Patel & Chellappa, 2013). Evidently, this is a NP-hard problem and solving it for large scale problems is impractical. In practice we seek to solve a reasonable approximation of this problem, where a small degree of inaccuracy is traded for large reductions in computational loads. In the next sections, some of these approximations are introduced and applied to the problem.

Application of CS Theory to the Moisture Estimation Problem

As discussed previously CS theory is only applicable for estimation of sparse/compressible signals. Obviously, moisture data over a field is not a sparse signal. Thus CS theory cannot be directly applied to the moisture estimation problem. It is however possible to apply CS theory to a modified form of the problem as shall be demonstrated next.

Important factors that mostly effect moisture content are precipitation, topography, soil properties, soil depth and vegetation (Gwak & Kim, 2016). Most of these factors do not change rapidly and can be considered almost constant over reasonable periods of time. This stationary feature means that while the absolute value of the soil moisture changes in time, the relative moisture between two points is predictable and changes much more slowly. In other words, soil moisture data is spatially correlated (X. Wu et al., 2011). Therefore, although moisture data is not a sparse signal itself, it can be transformed into the frequency domain using linear transformation such as DCT (Discrete Cosine Transformation) or DFT (Discrete Fourier Transformation) and in that domain, it will represent a sparse signal.

Let **x** be the moisture data vector at *N* locations that should be estimated using only *M* measurements. Since the problem of optimal sensor placement (optimal selection of *M* measurements from *N* points) is out of scope of this paper, simply assume that *M* measurements are randomly selected from the *N* points. Suppose that $\boldsymbol{\Phi}$ is the measurement matrix with *M* rows and *N* columns such that entries of each row contains *N*-1 zeros and 1 one. Hence, the measurement vector **y** is achieved by,

$$y = \Phi x. \quad (4)$$

Since **x** is not sparse, the DCT transformation is used to transform **x** to frequency domain. Let **Ψ** be the IDCT (Inverse Discrete Cosine Transformation) matrix. Accordingly, the new vector **α** can be defined such that,

$$x = \Psi \alpha. \quad (5)$$

Indeed, **α** is the transformed moisture data in the Fourier domain and therefore, it is a sparse signal. Thus **Eq. 3** can be reformed as,

$$\hat{\alpha} = \arg\min_{\alpha} \|\alpha\|_0 \text{ subject to } \ y = \Phi\Psi\alpha, \quad (6)$$

where,

$$\hat{x} = \Psi\hat{\alpha}. \quad (7)$$

As mentioned previously, M and $\boldsymbol{\Phi}$ must satisfy some constraints in CS theory. In approximation of the problem with the l_1-norm, M should be,

$$M \geq CK\mu^2(\Phi,\psi)\log N, \tag{8}$$

where C is a small constant, K is the number of nonzero elements of $\boldsymbol{\alpha}$ and $\mu(\Phi,\psi)$ is defined as,

$$\mu(\Phi,\Psi) = \sqrt{N}\max_{1\leq i,j\leq N}\left|< \Phi_i, \Psi_j >\right|. \tag{9}$$

Eq. 8 states that the minimum number of sensors for fine approximation of N points depends on the sparsity of the signal and the incoherency between $\boldsymbol{\Psi}$ and $\boldsymbol{\Phi}$. Fortunately, it can be shown that random selection of the measurement matrix $\boldsymbol{\Phi}$**,** makes it wholly incoherent with the Fourier basis matrix $\boldsymbol{\Psi}$ (Candes & Wakin, 2008). That it,

$$\mu(\Phi,\Psi) = 1. \tag{10}$$

Thus, the minimum number of sensors that are required for fine approximation depend on the sparsity of the signal.

Provided the above are satisfied, the moisture estimation problem becomes completely compatible with CS theory. In the next sections, various methods for approximation and solving the optimization problem will be introduced and applied.

Data Sets for Numerical Experiments

To solve the moisture estimation problem, a data set with sufficiently good resolution is required. Unfortunately, most real moisture data recordings that exist, are not at the required resolution or do not correspond to large fields. For this reason, data that is used in this paper is generated by state-of-the-art *TIN-based Real-time Integrate Basin Simulator (tRIBS)*. This simulator performs distributed hydrogeomorphic simulations over complex basins using Triangulated Irregular Networks (TIN) to form the basis for multiple-resolution representations (Vivoni, Teles, Ivanov, Bras, & Entekhabi, 2005).

Data that is used in this paper is a simulation of the Peacheater Creek Watershed. Peacheater Creek watershed covers an area of 64 km^2 and is located in the northeastern corner of Oklahoma. A simple map of this location is shown in **Fig 1** (S. Wu, Li, & Huang, 2007). Data is related to the conditions of summer 1991 and contains soil moisture values at the depth 100 mm at 6095 points of the field at 80 hours after start of the simulation.

In compressive sensing it is always desirable to have signals with higher degrees of sparsity. A simple way to increase sparsity in the frequency domain is to increase the correlation of data in the time domain by sorting them in an ascending order. The assumption of exactly sorting the moisture values is not practical, since if all moisture values are known a priori, estimation of moisture is no longer required. In practice, we derive a sorting index based on existing measured data, and use this index to sort all future values. This means data will not be exactly sorted, instead they will be approximately sorted in ascending order. The variations will show as high frequency low power harmonics which are negligible. A more detailed discussion on methods for robust sorting of field data is out of scope of this paper. For instant, coarse-grained monotonic ordering is a good method (X. Wu et al., 2011).

Another source of data variations are the measurement noises which corrupt true moisture values. If the levels of such noises exceed a certain amount when compared to the signal levels, the solution requires a robust compressive sensing approach. Basis Pursuit DeNoising (BPDN) formulation (Chen, Donoho, & Saunders, 1998) and solving it with iterative thresholding algorithms (Blumensath & Davies, 2008) is also a good approach for such situations.

The entire data set that containing sorted values at 6095 points is shown in **Fig 2**.

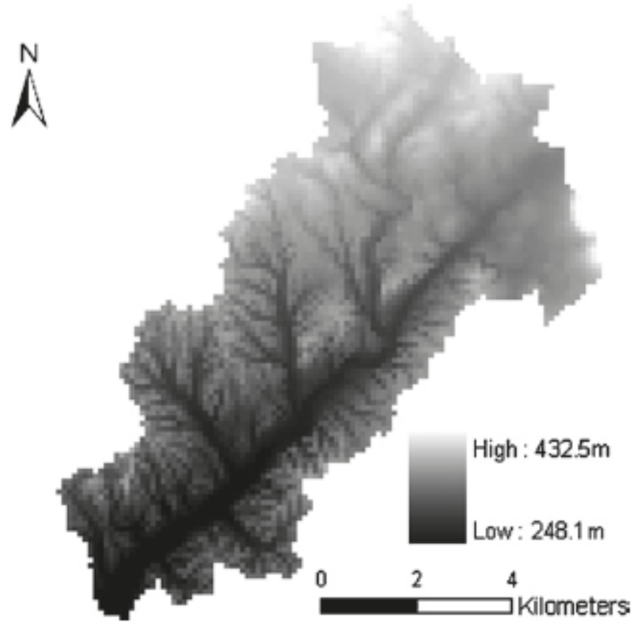

Fig 1. Peacheater Creek Watershed

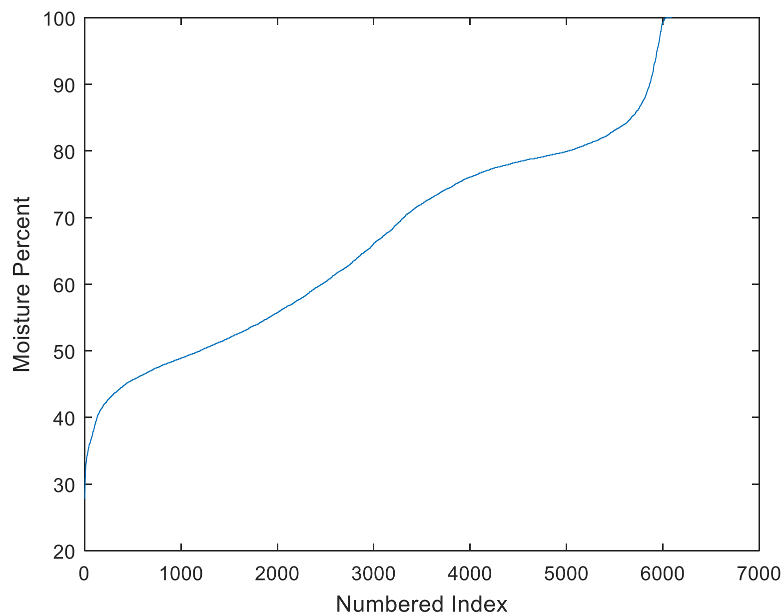

Fig 2. Whole data set at 6095 points sorted in increasing order.

Solving Optimization Problem Using Various Approximations

In this chapter, various approximations for solving the optimization problem in **Eq. 6** are introduced.

Simple l_1-norm Approximation

Many CS problems can be solved properly with simple l_1-norm approximation (Donoho, 2006). The formulation of the problem is similar to **Eq. 6** as follows,

$$\hat{\alpha} = \arg\min_{\alpha} \|\alpha\|_1 \text{ subject to } y = \Phi\Psi\alpha. \tag{11}$$

Eq. 11 is a convex linear problem and easy to solve.

Weighted l_1-norm Approximation

While in many cases, accurate results can be achieved by using simple l_1-norm approximation, in some instances, the l_1-norm optimization will produce totally false results when compared to the true l_0-norm solution. For example, consider $\mathbf{x}=[0\ 1\ 0]^T$, the measurement matrix,

$$\Phi = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 1 & 2 \end{bmatrix},$$

and the measurement vector $\mathbf{y}=\boldsymbol{\Phi}\mathbf{x}=[1\ 1]^T$. Solving the problem using l_1-norm approximation gives $\hat{\mathbf{x}}=[1/3\ 0\ 1/3]^T$ which is evidently a bad estimation (Candes et al., 2008). This simple example serves to illustrate that always using the l_1-norm is not a good approach for the relaxation of original problem.

The main difference between l_1-norm and l_0-norm is that in the l_1-norm, the magnitude of the nonzero elements is considered, but the l_0-norm only accounts for the number of nonzero elements. One appropriate approach to make l_1-norm a more accurate approximation for l_0-norm is to add weights such that the gap between the two norms may be arbitrarily reduced. With weights, **Eq. 3** becomes,

$$\hat{x} = \arg\min_{x} \|Wx\|_1 \text{ subject to } \ y = \Phi x, \tag{12}$$

where,

$$w_i = \begin{cases} \frac{1}{|\hat{x}_i|}, & \hat{x}_i \neq 0 \\ \infty, & \hat{x}_i = 0 \end{cases}. \tag{13}$$

If the above example is solved using **Eq. 13** with $\mathbf{W}=\mathrm{diag}([3\ 1\ 3]^T)$, then the result will be $\hat{\mathbf{x}}=[0\ 1\ 0]^T$ which is precisely correct. The large entries in w_i allow the solution $\mathbf{x}$ to take up larger values coinciding with the indices where w_i is small. Thus, the weighted l_1-norm approximation behaves like a l_0-norm. It is of course impossible to construct $\mathbf{W}$ accurately because it depends on the solution of the problem which is not known a periori. This means, solving the **Eq. 12** directly without knowing true values for $\mathbf{x}$ is not possible (i.e. the solution is needed to formulate the problem). To overcome this, iterative methods are used to solve the problem via weighted l_1-norm approximation (Candes et al., 2008). A popular iterative algorithm which is implemented in this paper for comparison is the following (Candes et al., 2008):

1. Set $w_i^{(0)}=1$, for $i=1,\ldots,n$.
2. Solve the weighted l_1 minimization problem:
$$x^{(l)} = \arg\min_{x} \left\|W^{(l)}x\right\|_1 \text{ subject to } \ y = \Phi x. \tag{14}$$
3. Update the weights:
$$w_i^{(l+1)} = \frac{1}{\left|x_i^{(l)}\right| + \varepsilon}. \tag{15}$$
4. Terminate on convergence or if l reach to specific number. Otherwise, increment l and go to step 2.

The value of ϵ in step 3 should be chosen to be slightly smaller than the expected nonzero magnitudes of $\hat{\mathbf{x}}$. Generally, the recovery process tends to be reasonably robust to the choice of ϵ. The moisture estimation problem in this paper is solved using this approximation with ϵ=5. The results will be shown in the next section.

FOCUSS Algorithm

One traditional method for reconstructing signals from some known values according to linear equation **Eq. 4** is to solve l_2-norm minimization problem,

$$\hat{x} = \arg\min_{x} \|x\|_2 \text{ subject to } \ y = \Phi x. \tag{16}$$

The solution of **Eq. 16** is unique and is computed as,

$$\hat{x} = \Phi^{+} y, \tag{17}$$

where Φ^{+} denotes the Moore-Penrose inverse (Campbell & Meyer, 2008). Although the solution has some computational advantages (the it has a closed for solution), but it does not provide sparse solutions. Indeed, it has a tendency to spread the energy over a large number of entries of **x** instead of putting all the energy into a few entries. Thus, it cannot be a good solution in reconstruction of sparse signal with low measurements. One modified algorithm of this simple solution, is FOcal Underdetermined System Solver (FOCUSS) (Gorodnitsky & Rao, 1997). FOCUSS algorithm is based on a weighted minimum norm solution as follows,

$$\hat{x} = \mathrm{W} \arg\min_{q} \|q\|_2 \text{ subject to } \; y = \Phi W q. \tag{18}$$

By changing **W** all possible solutions of the problem can be generated. If **W** is chosen properly, **Eq. 18** can be used for sparse signal reconstruction. Similar to weighted l_1-norm algorithm, FOCUSS algorithm proposes an iterative procedure for finding a suitable **W** and the recovery of the sparse signals.

The basic form of the FOCUSS algorithm is as follows,

1. For initialization, find $\mathbf{x_0}$ according to **Eq. 17**.
2. Compute weighting matrix:

$$W_{pk} = \left(diag(x_{k-1})\right). \tag{19}$$

3. Compute $\mathbf{x_k}$:

$$x_k = W_{pk}\left(\Phi W_{pk}\right)^{+} y. \tag{20}$$

4. Increment *k* and repeat steps 2 and 3 until convergence occurs.

Orthogonal Matching Pursuit (OMP) Algorithm

Another set of algorithms used for sparse signals reconstructions are greedy algorithms. There are several greedy algorithms for sparse recovery such as matching pursuit (Mallat & Zhang, 1993), orthogonal matching pursuit , gradient pursuits (Blumensath & Davies, 2009), regularized orthogonal matching pursuit (Needell & Vershynin, 2010) and stagewise orthogonal matching pursuit. The Orthogonal Matching Pursuit (OMP) algorithm (Tropp & Gilbert, 2007) is implemented in this paper for comparison purposes.

To recover the sparse signal **x**, in **Eq. 3** we need to determine which columns of $\boldsymbol{\Phi}$ participate in the measurement vector y. The main idea of the OMP algorithm is to pick columns in a greedy manner. At each iteration, the column of $\boldsymbol{\Phi}$ which most strongly correlates with the remaining part of **y** is selected. Then its contribution to **y** is subtracted and the process is repeated on the residual. If the main signal is *K*-sparse, after *K* iterations the algorithm will recover the signal properly.

The OMP iterative algorithm is as follows,

1. Initialize the residual $r_0 = y$, the index set $\Lambda_0 = \emptyset$, the matrix of chosen atoms $\Phi_0 = \emptyset$, and the iteration number *t*=1.
2. Find the index λ_t by solving following simple optimization problem:

$$\lambda_t = \arg\max_{j=1,\dots,d} \left|< r_{t-1}, \varphi_j >\right|. \tag{21}$$

3. Augment the index set and the matrix of chosen atoms,

$$\Lambda_t = \Lambda_{t-1} \cup \{\lambda_t\}. \tag{22}$$

$$\Phi_t = [\Phi_{t-1} \; \varphi_{\lambda_t}]. \tag{23}$$

4. Solve a least squares problem to obtain a new signal estimate,

$$s_t = \arg\min_{s} \|y - \Phi_t s\|_2. \tag{24}$$

5. Calculate the new approximation of the data and the new residual,

$$\alpha_t = \Phi_t s_t. \quad (25)$$
$$r_t = y - \alpha_t. \quad (26)$$

6. Increment t and return to step 2 if $t < K$.
7. The estimate $\hat{\mathbf{x}}$ has nonzero indices at the components listed in Λ_K. The value of the estimate $\hat{\mathbf{x}}$ in component λ_j equals the *j*th component of s_t.

Note that the estimation problem in our case is solved for the transformed data in frequency domain, not the moisture values themselves. Regarding DCT coefficients of the moisture data, the number of coefficients that are necessary for reconstructing signal in our case, is about 40. Hence the problem is solved using *K*=40.

Comparison of Different Algorithms

To compare results of different algorithms, *M*=200 sensors are selected randomly and the same sensors are used for all methods. The estimated moisture values using the reviewed methods are presented in **Fig 3**. As it can be seen, the OMP algorithm delivers the best performance and the FOCUSS algorithm performs rather poorly. This was somewhat predictable since the unweighted l_1-norm minimization encourages sparsity, while unweighted l_2-norm discourages it to some extent. Clearly the added weights have not helped greatly in reducing the gap with the l_0-norm, or even the l_1-norm solution.

To obtain more insight into the problem, the estimation problem is re-computed for varying number of sensors from 50 to 350 in 10 sensor increments. We introduce two key performance indicators (KPI) for comparison of the performance of the algorithms. These are the RMSE and recovery percent.

The Root Mean Square Error (RMSE) is defined as follows,

$$RMSE = \frac{\|\hat{x}-x\|_2}{\sqrt{N}}. \quad (27)$$

The second criterion is considered as the ratio of the values that are recovered correctly to the number of all values. A value is assumed correctly recovered if the error between the estimated value and the real value is below %1.

The results of using different algorithms with varying number of sensors are shown in **Fig 4** and **Fig 5**. These plots confirm that the FOCUSS algorithm is not appropriate for the moisture estimation problem. Another important point that should be noticed is that the main difference between the algorithms is becomes apparent when using a few number of sensors. When permitted sensor allocations are large enough, the results of remaining three algorithms are nearly equivalent. For instance, if 500 sensors are allocated, there is no meaningful difference between the performance of the algorithms.

It is indeed difficult to conclude that one method universally outperforms the others. The actual location of the sensors also seems to sometimes have an opposing effect on the performance of the algorithms. For example in the case of using 170 sensors, recovery percent with weighted l_1-norm is higher than OMP, while by using 180 sensors in the next step, the results are completely inverse (see **Fig 4**). Note that in the case of using 180 sensors, the results of weighted l_1-norm get worse than 170 sensors, while the results of OMP get better. Since sensor placement is allocated randomly, it is natural not to see a monotonic increase in performance, but the significant change in recovery percentages with a change in the locations was less expected, illustrating that the different estimation algorithms have different sensitivity to the location of the sensors. This also serves to illustrate the fact that in such estimations problems, the location, as well as the number of the sensors are both very important for successful estimation.

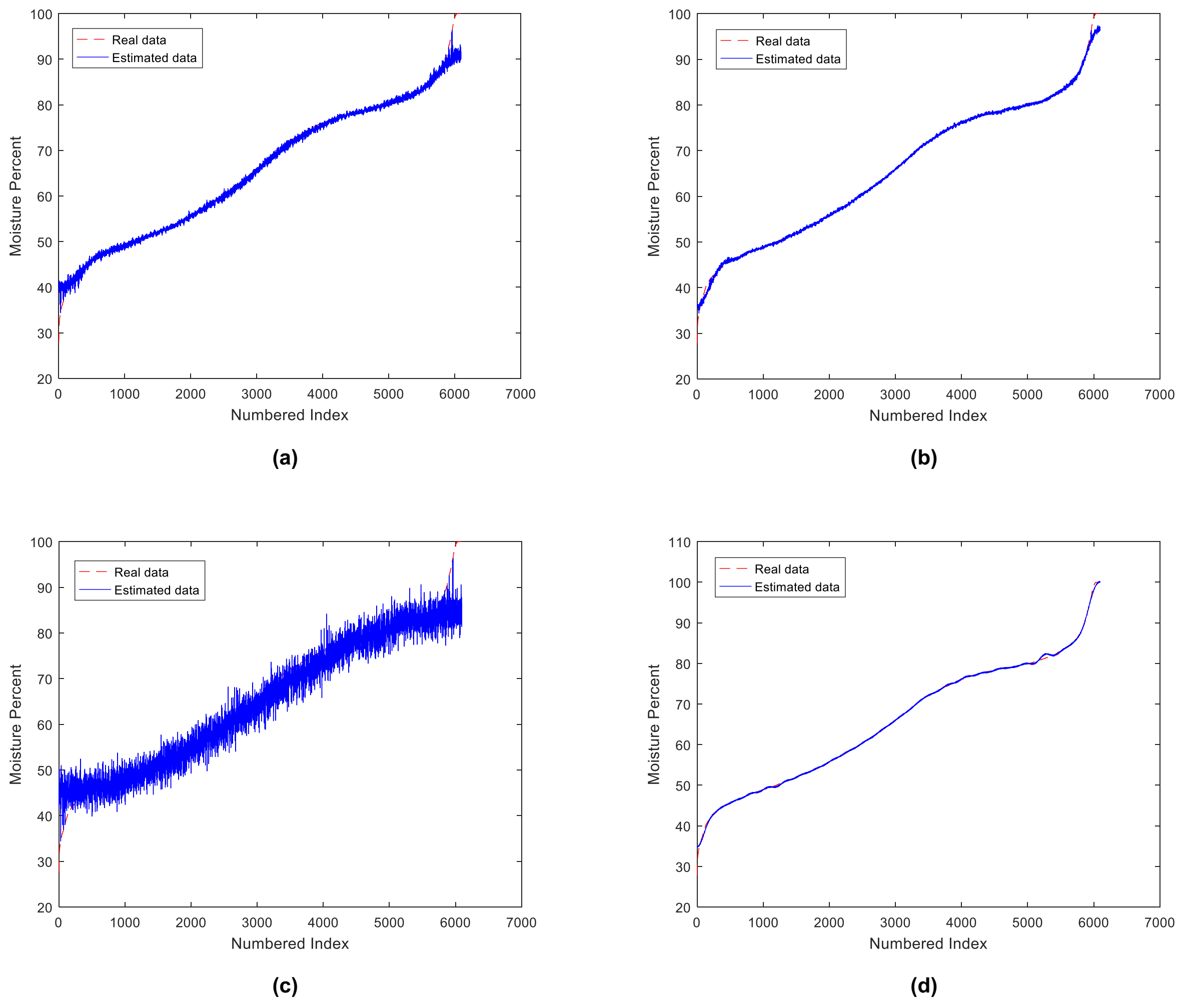

Fig 3. Estimated moisture data using different approximation methods using 200 sensors with random placement. (a) l_1-norm. (b) Weighted l_1-norm. (c) FOCUSS. (d) OMP.

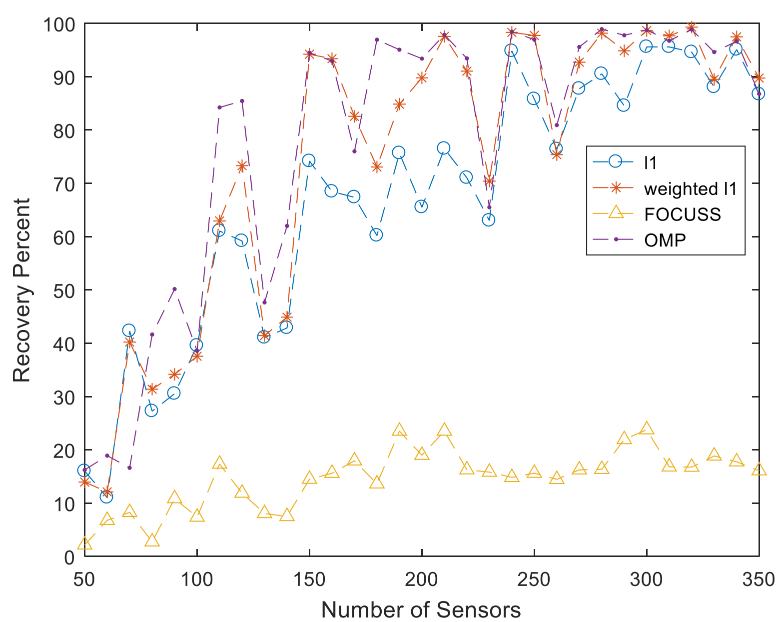

Fig 4. Comparison of different algorithms using different number of sensors with criterion recovery percent.

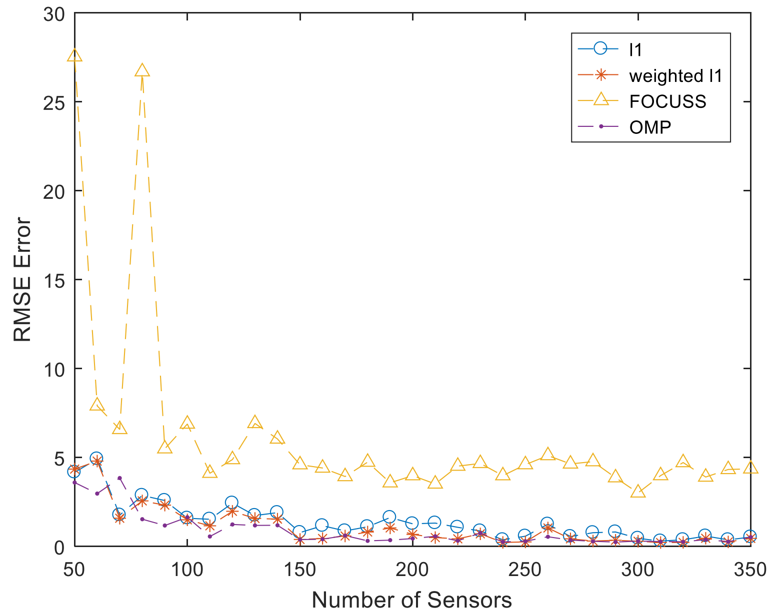

Fig 5. Comparison of different algorithms using different number of sensors with criterion RMSE.

To verify the effect of the random sensor selection of the location of the sensors on the results, consider the cumulative performance of the algorithms over 4 runs, each of which has a random allocation. The problem is solved 4 times by each algorithm for sensors number *M* from 50 to 200 in 10 sensor increments and the mean of results is shown in **Fig 6** and **Fig 7**. As can be seen the cumulative performances converge to a monotonically increasing trend which is as expected. If the average of an infinite number of runs is computed, the results would become exactly monotonically increasing.

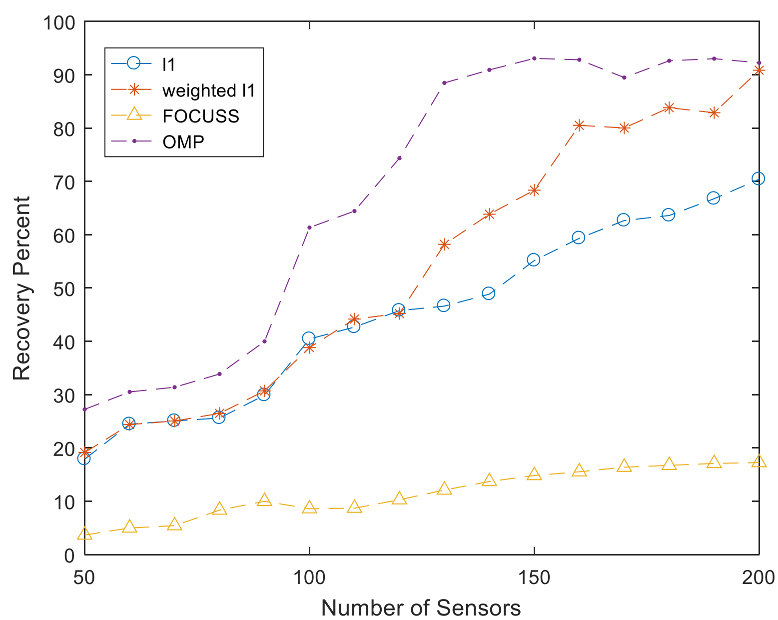

Fig 6. Comparison of different algorithms using different number of sensors with criterion recovery percent. To show overall preference of OMP algorithm in a better way, the problem is solved four times by each method and mean of the results is shown.

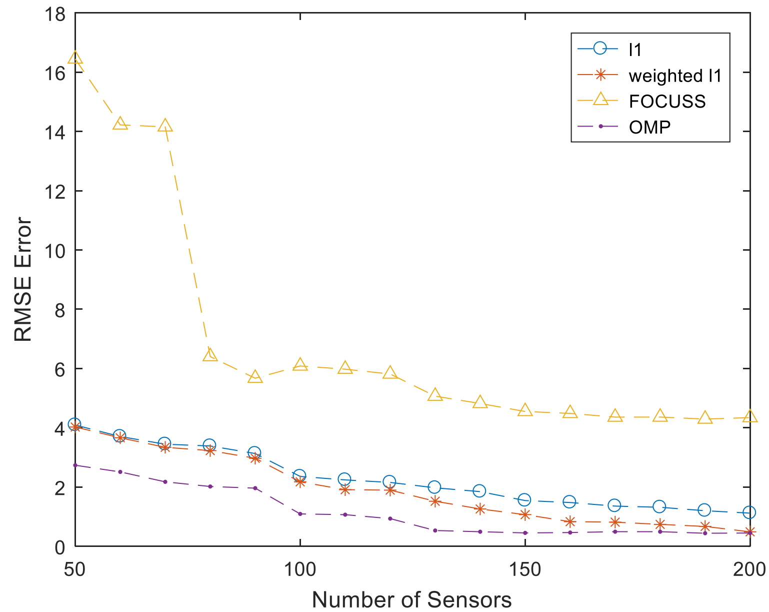

Fig 7. Comparison of different algorithms using different number of sensors with criterion RMSE. To show overall preference of OMP algorithm in a better way, the problem is solved four times by each method and mean of the results is shown.

Another criterion that can be considered in choosing the proper method is computational time. The computational effort will gain significance in very large estimation problems, or when estimation has to be performed in embedded or low computational power industrial processors. In online estimation and control problems this will also become a consideration. We compare the time required to solve the problem by using *M*=200 sensors in **Table 1**. All computations are done using *MATLAB R2016b* by a PC with processor AMD E1-6010 APU with 4 GB RAM. All optimizations are solved using solver *cvx*.

Table 1. Comparison of computational time of different methods.

Method	Computational time in seconds
l_1-norm	80.6
Weighted l_1-norm	787.8
FOCUSS	36.4
OMP	70.0

When considering the performance and computational cost of the algorithms, it seems that for real life moisture estimation problems, the OMP algorithm offer a better overall package.

Conclusion

Compressive sensing theory was formulated for the moisture estimation problem. Since the main problem contains a l_0-norm optimization that is not practical to solve, four approximations were reviewed and applied. The results of the recovered signals using different methods with different number of sensors were compared. As far as it appeared from the results of these comparisons, the OMP algorithm seems to be a better overall choice for the moisture estimation problem.

The results of this paper are based on well sorted data and noiseless measurements with random sensor placement assumption. The effect of noisy and corrupt measurement needs to be investigated further. It is also necessary to perform the study for fields with different topologies. It may be that for different geographical conditions, the results of the comparison would alter markedly.

Finally, it should be investigated to see if it better to consider moisture values as a 1-D vector and try to sort them for enhancing sparsity or consider the field as a 2-D matrix like a picture and use natural geographically correlation of them and apply CS theory in 2-D.

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