Flat payoff functions and site-specific crop management

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Abstract. *Within the neighbourhood of any economically "optimal" management system, there is a set of alternative systems that are only slightly less attractive than the optimum. Often this set is large; in other words, the payoff function is flat within the vicinity of the optimum. This has major implications for the economics of variable-rate site-specific crop management. The flatter the payoff function, the lower the benefits of precision in the adjustment of input rates spatially within a crop field. This paper is about how we can best measure the flatness of payoff functions, in order to assist with judgments about the likely benefits of site-specific crop management. We show that two existing metrics — the relative range of an input for which the payoff is at least 95% as large as the maximum payoff (IR95) and the relative curvature (RC) of the payoff function — are flawed. We suggest an alternative metric: the standard deviation of the slopes of site-specific payoff-functions at the optimal uniform input rate (SDS).*

Keywords. *Payoff function, curvature, nitrogen managment*

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Introduction

Payoff functions for agricultural inputs[1] are generally flat in the vicinity of the optimal input rate (Pannell 2006). In other words, at input rates somewhat above or below the optimum, the payoff to farmers is only slightly less than the payoff at the optimal input rate. Pannell (2006) showed that this has a range of implications for the economics of farm management, including for precision technologies that allow site-specific crop management. The flatter the curve, and the wider the input range over which it is flat, the lower the benefit from adjusting input rates spatially in response to local conditions.

Although flat payoff functions are the norm, the flatness of the curve varies to some degree from case to case. This suggests a strategy of measuring the degree of flatness of payoff curves, in order to identify situations where the benefits of site-specific crop management are most likely to be high. Information about flatness may contribute to decisions by farmers about their investment in precision technologies, may assist precision-technology researchers to target their efforts to the most promising contexts (e.g. regions, crops or soil types), or may assist technology sellers to target their sales activities to contexts where they are most likely to succeed.

Our aim in this paper is to evaluate options for measuring the flatness of payoff functions, in the hope of identifying a simple option that reflects the economic benefits of site-specific crop management reasonably accurately. Two options have already been used in the literature. Pannell (2006) used a simple but *ad hoc* indicator of flatness: the range of input level (x) for which the payoff $\pi(x)$ is at least 95% as large as the maximum payoff $\pi_{max} = \pi(x^*)$, where x^* is the input level that maximises the payoff function. Although this was not proposed as an indicator for decision making, it could potentially be used as such. Normalised to x^*, this indicator is:

$$IR95 = \frac{x_u - x_d}{x^*} \qquad (1)$$

where *IR95* stands for the Input Range 95, $\underline{x_u}$ is the upper limit of the range of input levels that result in a payoff of at least $0.95 \times \pi_{max}$, and x_d is the downside limit of that range. The variables used to calculate IR95 are shown in Figure 1.

[1] e.g. the relationship between an input rate and profit per unit area, or between input rate and expected utility.

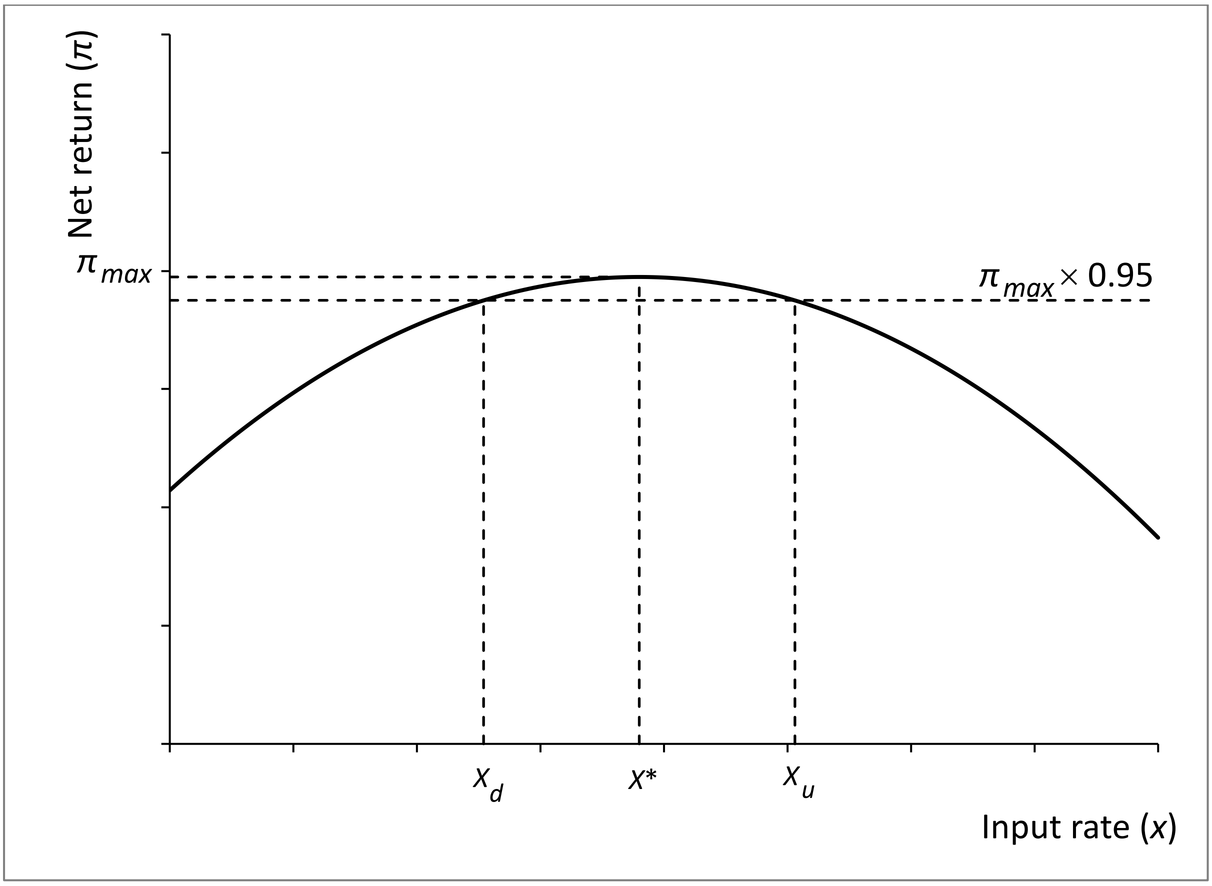

Figure 1. An example payoff function showing the range of input levels giving a payoff at least 95% of the maximum payoff, π_{max}.

Rogers et al. (2016) proposed a second measure of the flatness of a payoff curve, which they termed relative curvature (RC).

$$RC = \frac{\pi_{max} \times \hat{x} - \int_0^{\hat{x}} \pi(x) dx}{\pi_{max} \times \hat{x}} \tag{2}$$

where $\hat{x}$ is an arbitrary input level that sets the upper range for measuring RC. The calculation of RC is illustrated in Figure 2. It is equal to the shaded area divided by the area of the rectangle $\pi_{max} \times \hat{x}$.

Rogers et al. found that relative curvature varies substantially between cases, and suggests that it be used to identify those fields where site-specific crop management should be applied. They also showed that RC increases when external environmental costs are internalised (e.g. a pollution tax is levied on farmers for each unit of a nutrient that leaves their property). They interpreted this as meaning that site-specific crop management is more beneficial when the environmental impacts of nutrient use are accounted for.

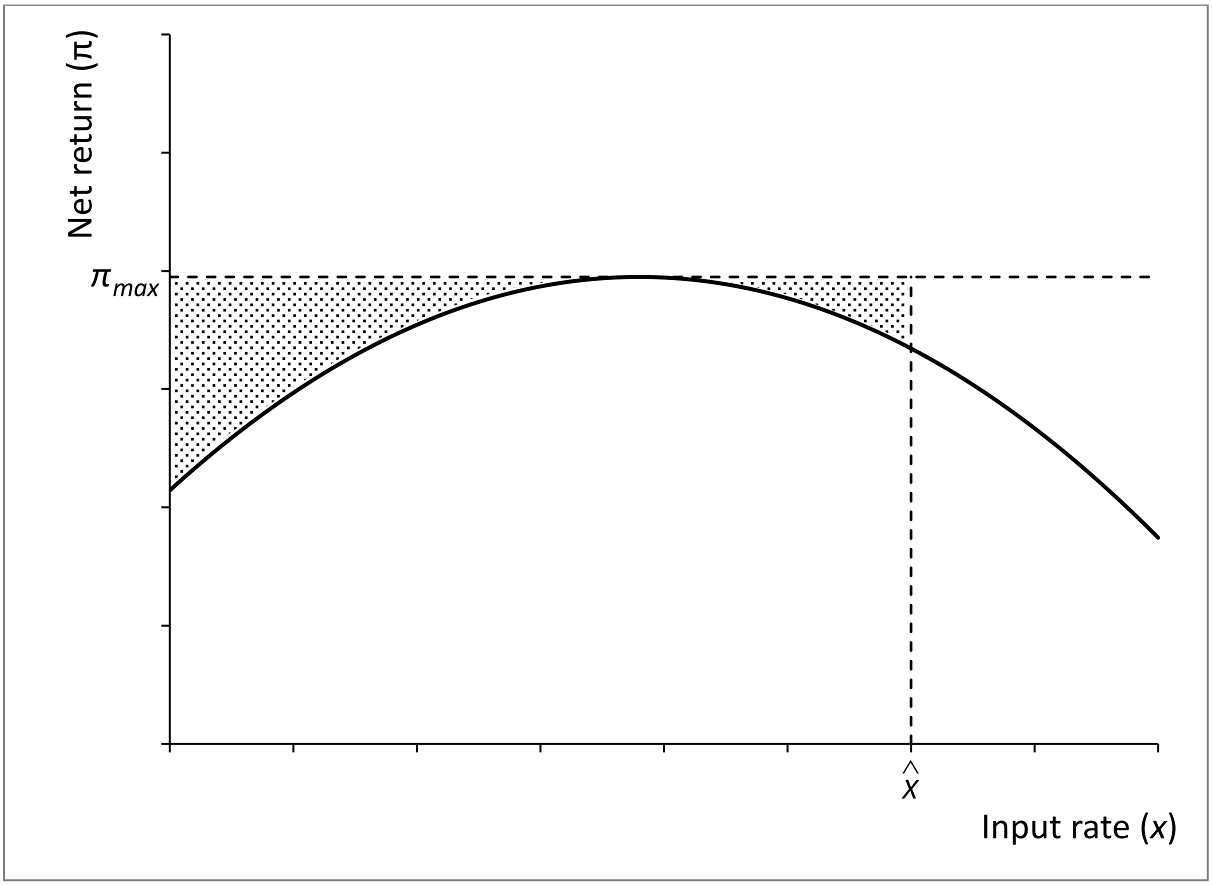

Figure 2. The Relative Curvature of the payoff function is defined as the shaded area divided by the area of the rectangle $\pi_{max} \times \hat{x}$.

In this paper we evaluate the usefulness of these two measures of flatness. Rogers et al. (2016) argued that RC is superior to IR95, but they did not compare the economic performance of each when used as a guide to decision making. In addition, we evaluate a third measure: the standard deviation of the slopes of site-specific payoff-functions at the optimal uniform input rate, x^* (SDS for short). This measure recognises that there are different payoff functions in different areas of a field. We assume that the field can be broken into *N* areas, each of which is uniform within that area. Each area has a different payoff function to the input.

The overall payoff function for a uniform input rate is the weighted combination of the payoff functions for the *N* parts of the field. From that overall payoff function, we determine the optimal uniform input rate for the field, x^*. Then, for each part of the field, we determine the slope of the payoff function at x*, which, for the *i*th part of the field, we represent as $\pi_i'(x^*)$. Then we calculated the standard deviation of those slopes across the field, *SDS*:

$$SDS = \sqrt{E\left[\pi_i'(x^*) - \overline{\pi_i'(x^*)}\right]^2} \tag{3}$$

where $\overline{\pi_i'(x^*)}$ is the mean slope of the payoff function across the *N* different areas of the field. The calculation of standard deviation is weighted by the area for which each payoff function applies. Figure 3 shows a simple illustrative example, where *N* = 3. The tangents at x* for each of the three payoff functions are shown. SDS is the standard deviation of the slopes of these tangents. The reason for testing this metric is that the slope of the payoff function at x* indicates the potential gain in payoff from adjusting the input rate away from the optimal uniform level, and that it reflects the heterogeneity of the field, which underpins the gains from site-specific management.

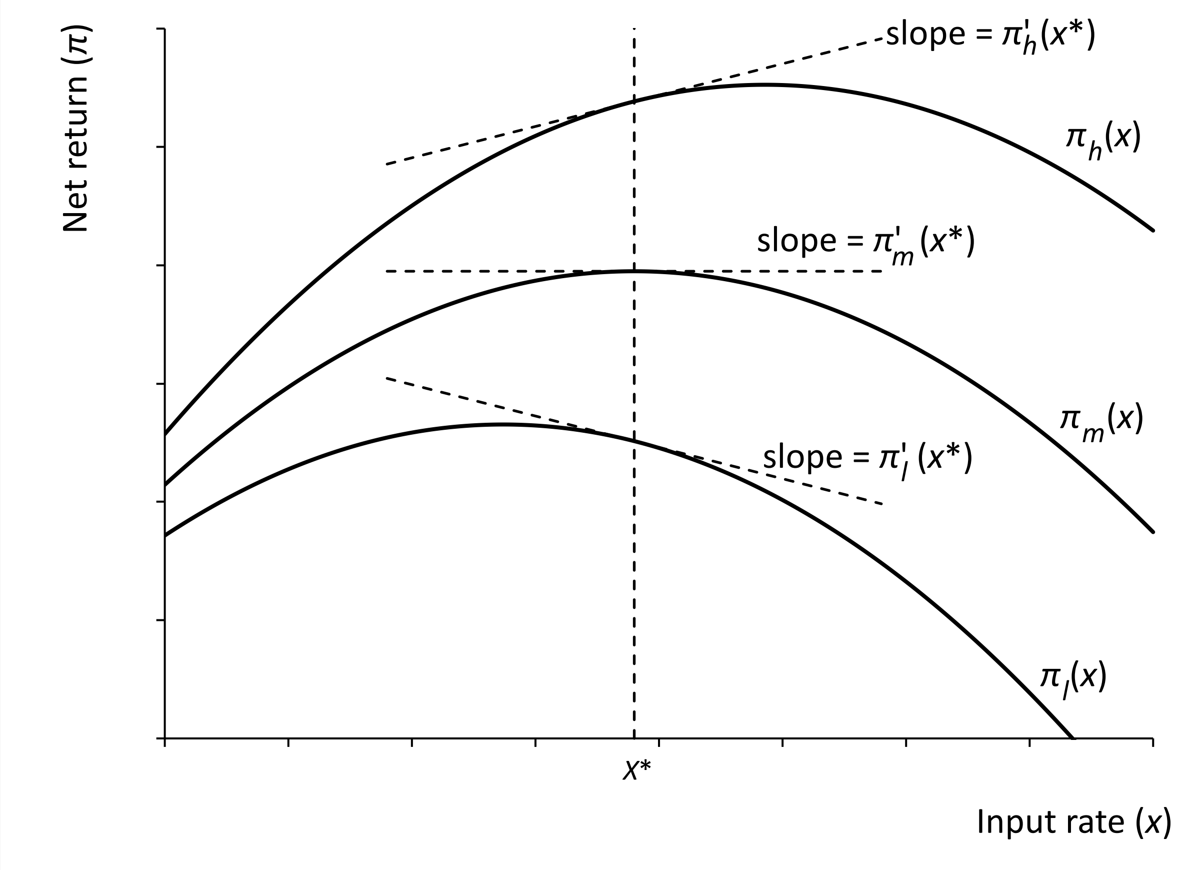

Figure 3. Illustration of the slopes used to calculate SDS for a case where there are three discrete regions in a field with payoff functions $\pi_l(x)$, $\pi_m(x)$ and $\pi_h(x)$.

Economic benefits of site-specific crop management

In order to evaluate the suitability of the three flatness metrics for indicating the economic benefits of site-specific crop management, an economic model of site-specific crop management is needed. We develop the model for the simple illustrative case where $N = 3$, but it is generalizable to any N.

The benefits of adjusting input rates spatially arise because different parts of a field have different payoff functions, resulting in different optimal input rates. Figure 4 shows how the optimal input rate varies for this example, where there are three discrete parts of the field. The middle payoff function is based on a function estimated by Meyer-Aurich et al. (2010) for nitrogen application to wheat in Germany. The other two curves have yields 20% higher and 20% lower than the central curve. For these three payoff functions, the optimal nitrogen rates are $x_l = 137$, $x_m = 190$ and $x_h = 243$ kg per hectare. On the basis of this wide variation in optimal rates, it might be expected that the benefits of adjusting nitrogen rates across the three parts of the field would be high.

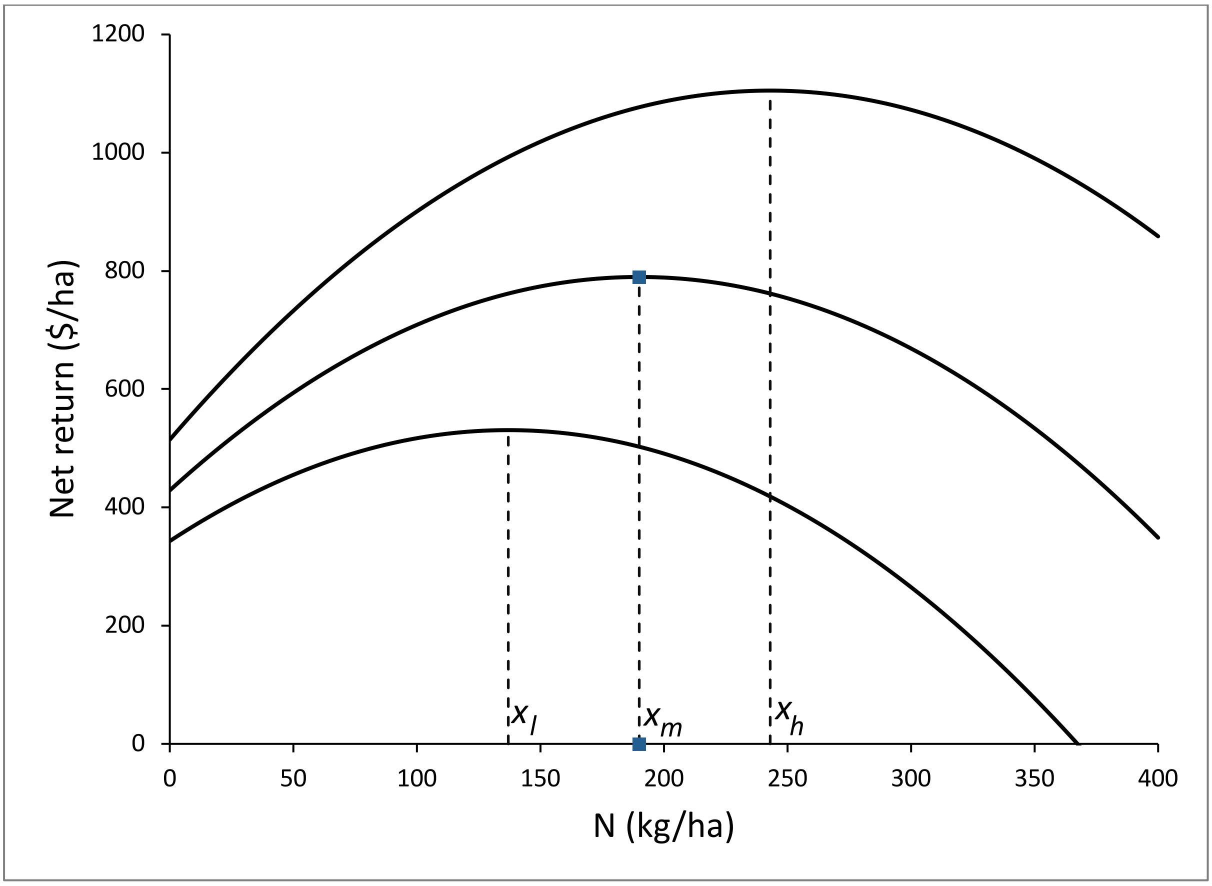

Figure 4.Optimal nitrogen fertilizer rates, x_l, x_m and x_h, for three payoff functions occurring within parts of a field.

Assuming that the field consists of 50% medium yield, 25% high yield and 25% low yield areas, the optimal N rate for a uniform application (x^*) in this case is 190 kg/ha, the same as for the medium yield area. Figure 5 shows the benefits in each area from undertaking site-specific crop management, compared to applying the optimal uniform fertiliser strategy x^* across the whole field. In the low-yielding area, the benefit of reducing the fertilizer rate from x^* to x_l equals *Pls* – *Plu*, the net return at x_l minus the net return at x^*. Because of the flatness of the payoff function, the proportional increase in net return is much less than the proportional reduction in input level. Similarly, the benefit of increasing the fertilizer rate from x^* to x_h equals *Phs* – *Phu*, the net return at x_h minus the net return at x^*. Again, the increase in net return is small relative to the increase in fertilizer rate. Finally, in the medium-yielding area, there is no gain in net return under site-specific management because, in this example, the optimal input rate for this area is the same as the optional uniform input rate, $x_m = x^*$ so *Pms* = *Pmu*.

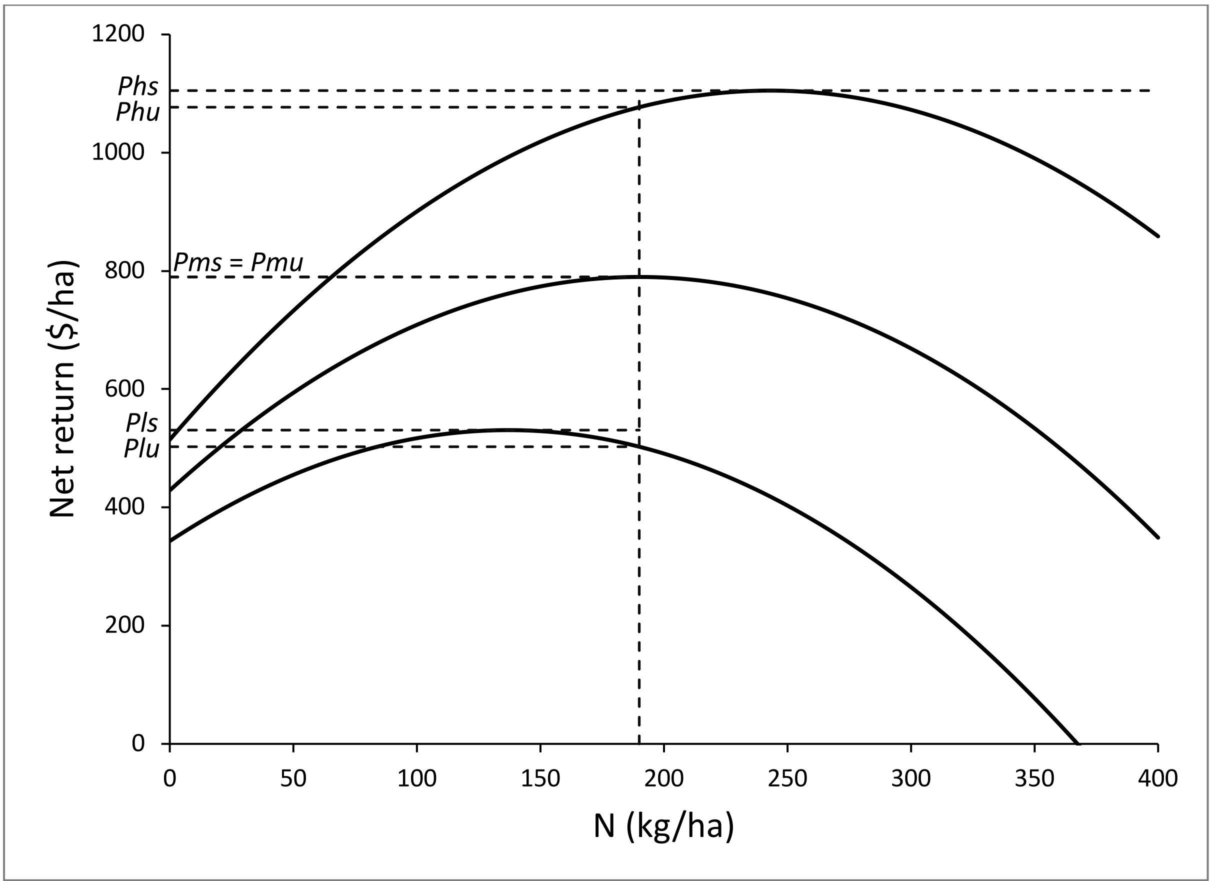

Figure 5. The benefits of site-specific crop management within three areas of a field with different payoff functions.

Given these profit improvements for each area, the overall profit improvement under site-specific crop management is calculated as follows. Firstly, the net return under site-specific crop management is

$$\pi_s = Phs \times A_h + Pms \times A_m + Pls \times A_l \tag{4}$$

where A_h, A_m and A_l are the areas of high-, medium- and low-yield land within the field.

Under the optimal uniform rate, net return is

$$\pi_u = Phu \times A_h + Pmu \times A_m + Plu \times A_l \tag{5}$$

The benefit (B) of site-specific crop management relative to uniform management is the difference between (4) and (5):

$$B = \pi_s - \pi_u = (Phs - Phu) * A_h + (Pms - Pmu) * A_m + (Pls - Plu) * A_l \tag{6}$$

This gives us our measure of the gross benefit of site-specific crop management relative to uniform rates. In evaluating the overall performance of site-specific management, extra costs would also have to be considered, but here we focus only on the benefit. We express this benefit relative to the maximum net return under site-specific management:

$$B_r = (\pi_s - \pi_u) / \pi_s \tag{7}$$

Results

For the base-case scenario, with three yield zones and a symmetrical distribution of yields within the field, the potential gain in profit from switching from a uniform fertilizer rate to a site-specific one is €14.0, or 1.8% (Table 1, first row of results). The flatness of the three payoff functions means that the changes in profit are much smaller percentages that the changes in fertilizer rates, as shown in Tables 4 and 5. The *IR95* indicator is 0.66, meaning that the range of input rates that give profits at least 95% of the optimal uniform rate is 66% of the optimal uniform rate. *RC* is 0.17 and *SDS* is 0.75 – values that that are not helpful in themselves but may be useful when compared across scenarios.

Table 1. The economic benefit of precision and the performance of the three metrics measuring flatness, for various scenarios and yield distributions.

Scenario	Distribution of Low-Med-High yield zones	Benefit of precisions (*B*) (€/ha)	Relative benefit of precision (B_r)	Input Range 95 (*IR95*)	Relative curvature (*RC*)	Standard dev'n of the slopes (*SDS*)[A]
Base case	25-50-25	14.0	0.018	0.66	0.17	0.75
High yield variance	25-50-25	31.6	0.040	0.66	0.17	1.12
Very high yield variance	25-50-25	56.2	0.071	0.66	0.17	1.50
Flatter payoff function	25-50-25	12.5	0.016	0.88	0.08	0.50
Flatter payoff function, high yield variance	25-50-25	28.1	0.036	0.88	0.08	0.75
Flatter payoff function, very high yield variance	25-50-25	49.9	0.063	0.88	0.08	1.00
Base case	33-33-33	18.7	0.024	0.66	0.17	0.87
High yield variance	33-33-33	42.1	0.053	0.66	0.17	1.30
Very high yield variance	33-33-33	74.9	0.095	0.66	0.17	1.74
Base case	25-25-50	19.3	0.022	0.65	0.15	0.88
High yield variance	25-25-50	43.5	0.048	0.64	0.15	1.32
Very high yield variance	25-25-50	77.3	0.082	0.64	0.14	1.76

[A] Standard deviation of the slopes of site-specific payoff-functions at the optimal uniform input rate.

The benefits of precision depend on the variability of yields across different zones of the field. The second and third sets of results are for the scenarios with high and very high yield variance (Table 1). The benefit of precision (*B* or B_r) increases with yield variance. This is illustrated in Figure 6, which is the equivalent of Figure 5 but for very high yield variance. A comparison of Figures 5 and 6 reveals the reasons for the effects of yield variance on the benefits of site-specific management. Higher yield variance means that optimal site-specific input rates are more variable and the slopes of the payoff functions at the optimal uniform rate are higher, meaning that input rate adjustments make a bigger difference to payoffs. Although the economic gains from precision are more than three times larger under the very-high yield-variance scenario compared with the base case, they are still relatively modest at 7.1%, reflecting the strong influence of payoff-function flatness.

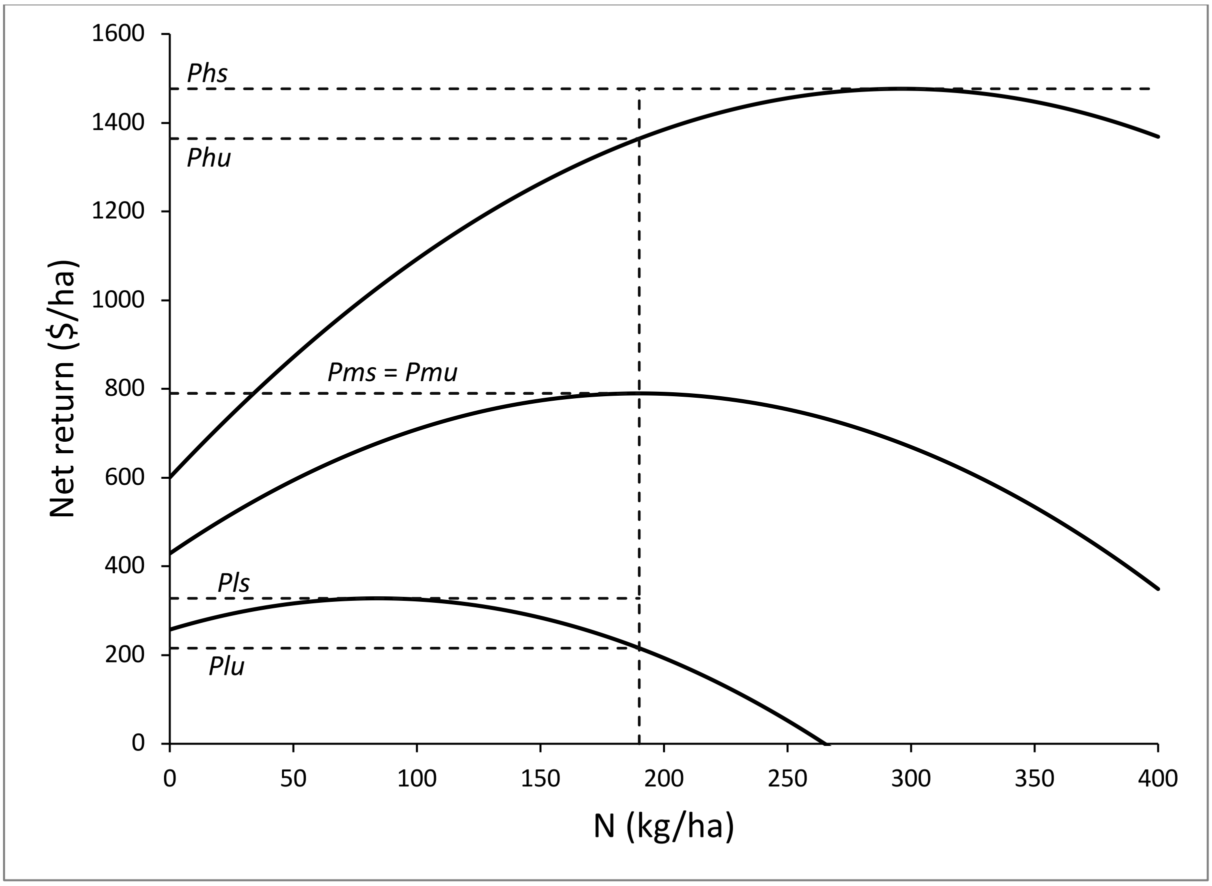

Figure 6. The benefits of site-specific crop management within three areas of a field with different payoff functions, under the very high yield variance scenario.

Of the three metrics of payoff-function flatness, only *SDS* reflects the increasing benefits of site-specific inputs under increasing yield variance (Table 1). Both *RC* and *IR95* are unchanged across the three yield-variance scenarios because both are calculated from the mean payoff function, which is unchanged across these scenarios. *SDS* is positively correlated, although not perfectly, with *B* and B_r.

The benefits of site-specific crop management also depend on the flatness of the payoff curves. The payoff curves in Figures 4 to 6 are not particularly flat compared to some examples (e.g. Pannell 2006). In Figure 7, the flatness is increased by halving the *c* parameter. The *a* parameter is increased to give the same yield in medium-yield zones at a nitrogen rate of 200 kg/ha. The range of yields is the same as for the base case. Because of the increased flatness, the benefits of site-specific management are reduced at each level of yield variability (Table 1) – by about 11% in each case.

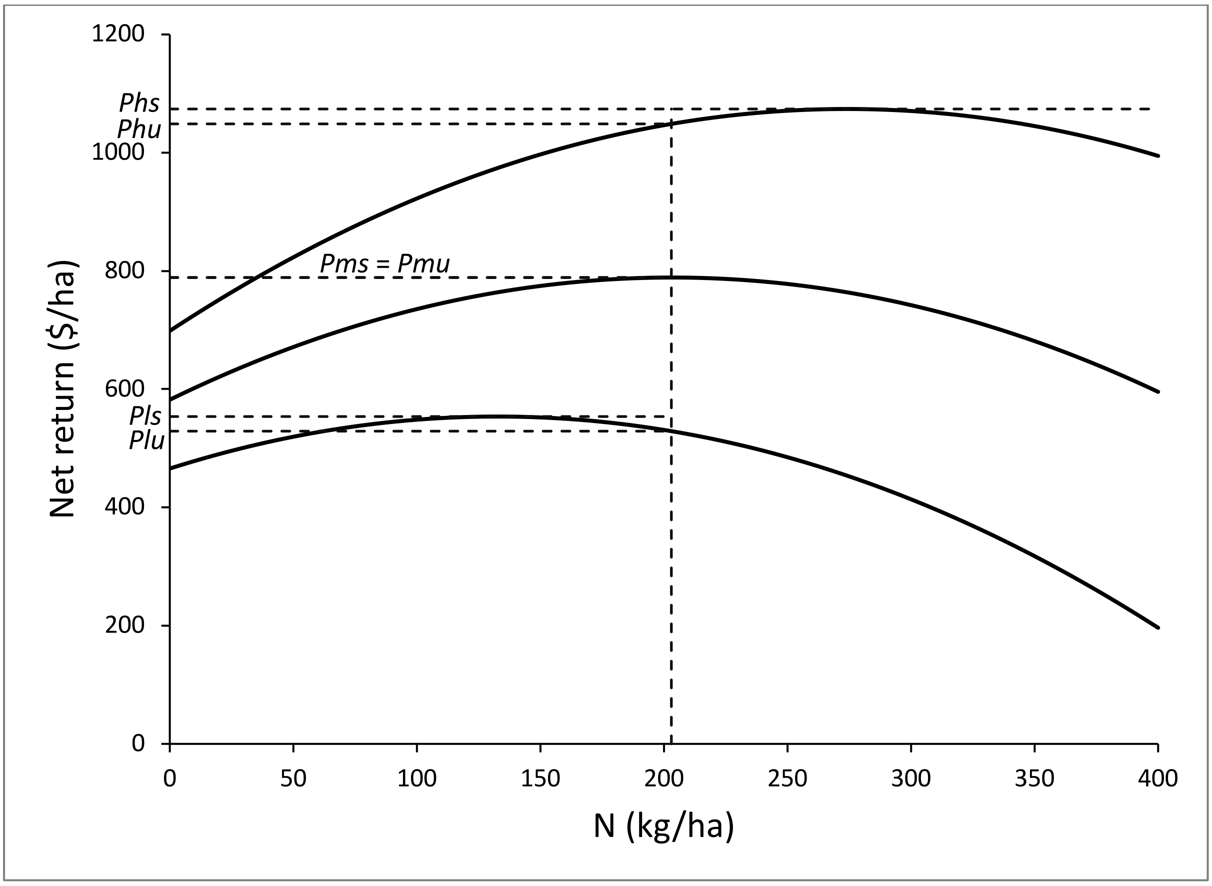

Figure 7. The benefits of site-specific crop management with relatively flat payoff functions.

All three of the indicators overstate the impact of flatness on the benefits of precision. *IR95* and SDS both change by 33% in response to the increased flatness, while *RC* falls by 51%, compared with the actual change in benefits of 11%.

Next we explore the shape of the distribution of yields. The base case has a symmetrical distribution of the areas of low, medium and high yielding zones (25, 50, 25%, respectively), which is realistic in many cases (Rogers et al. 2016). However, the empirical measurements presented by Rogers et al. also include different distributions in some cases. For this reason, we also simulate results for a uniform distribution (33, 33, 33%) and a skewed distribution (25, 25, 50%) (Table 1).

Both of these distributions give more weight to the high and/or low yielding zones, and as a result there are greater benefits from adjusting input rates away from the uniform rate. For the uniform distribution, the benefits rise by 33%, while for the skewed result they rise by almost 40%. (B_r rises by a smaller proportion because the skew towards high-yielding zones means that expected profit is higher, so the gain relative to expected profit is lower).

For the uniform distribution, *SDS* understates increase in benefits from precision relative to the symmetrical distribution. However, *RC* and *IR95* fail to detect any benefit at all. For the skewed distribution, the change in *SDS* relative to the base case is about half of *B*, while IR95 detects almost no benefit, and *RC* incorrectly indicates a reduction in benefits from precision.

Discussion

The benefits from site-specific management for the application of nitrogen to wheat are small; the relative increase in net returns range from 2% in the base case to approximately 9% in the extreme case with very high yield variance and an even distribution of land area across management zone types, although we consider the latter scenario to be unrealistic in practice. The results are consistent with the lack of adoption of precision agriculture technologies (OECD 2016). Increasing the degree of heterogeneity in the field, increases the benefits of site-specific management but the results suggests that the enhanced returns from precision are unlikely to cover the costs of the technology under many situations unless capital costs decrease significantly.

A major reason for the relatively small benefits from site-specific application of nitrogen is the flatness of the payoff curve showing the relationship between nitrogen use and the net returns from varying the rate by management zone. The previously used measures of flatness, *IR95* and *RC,* are highly unsuitable to use as indicators of the benefits of variable-rate site-specific crop management. *IR95* was not originally proposed as an indicator for decision-making. It may still be useful is in conveying the concept of the payoff curve being flat by highlighting the range of inputs for which net returns are only 5% less than the optimal.

The *SDS* is an indicator that does correlate highly with benefits from precision management; the higher the slope of the payoff function in each management zone, the greater the benefits from site-specific management. On the other hand, data requirements to apply the *SDS* indicator are high. As a minimum, it needs information about yields at a near-optimal input rate and another moderately different rate, for various areas in the field. If an analyst had sufficient information to calculate the SDS, it would be only a small step to calculate the economic benefits of variable-rate technology using the economic model presented earlier. Like *IR95*, its main contribution in practice maybe to influence perceptions and understanding.

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