

# An Economic-Theory-Based Approach to Management Zone Delineation

## Brittani Edge

Agricultural and Consumer Economics, University of Illinois Urbana-Champaign, Urbana, Illinois

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**Abstract.** In both the academic and popular literatures on precision agriculture technology, a management zone is generally defined as an area in a field within which the optimal input application strategy is spatially uniform. The characteristics commonly chosen to delineate management zones, both in the literature and in commercial practice, are yield and variables associated with yield. But microeconomic theory makes clear that economically optimal input application strategies do not necessarily depend on yield levels; rather, they depend on the responses of yields to inputs. Therefore using "yield zones" to determine "management zones" is likely to be a suboptimal strategy, and these zones should instead be delineated using characteristics that affect the yields' response to inputs. Specifically, a management zone should be an area of the field with the same marginal product function with respect to the input being managed.

This paper reports research that uses data from a 2017 full-field randomized agronomic trial to assess the impact on economic profits of these the yield-based and economic-theorybased approaches to nitrogen management zone delineation. The response of yield to many different factors, including managed inputs and soil and field characteristics (e.g., electroconductivity and slope), is known as a yield function, and the marginal product of nitrogen is the derivative of this function with respect to nitrogen. Estimation of the yield function for this field used a quadratic form, and considered both a full set of covariates containing results of a soil test and a subset of the covariates with only electroconductivity and topography variables. Results indicate a spatial error model is appropriate as shown by a significant nonzero Moran's I on the error terms from the OLS estimation. Additionally, results indicate there is information contained in the soil tests that impacts yield estimation but the value is not estimated in this preliminary work. The results of this research inform both future literature and commercial activity in precision agriculture. With the establishment of theoretically consistent new methods for delineating management zones, management advice may improve, and the value of data collection such as soil sampling and electroconductivity measurement can be increased.

## Keywords.

Management zones, soil tests, precision agriculture, corn yield estimation.

#### Introduction

In both the academic and popular literatures on precision agriculture technology, a management zone is generally defined as an area in a field within which the optimal input application strategy is spatially uniform. Despite major advances in and strong adoption of variable-rate technology, the delineation of management zones has remained largely unchanged. This research proposes and evaluates a new delineation method using economic principles that were previously ignored in the literature. The characteristics commonly chosen to delineate management zones, both in the literature and in commercial practice, are yield and variables associated with yield. These variables were selected because previous literature assumed an input should be applied more intensively on higher yielding parts of a field; however, it can be shown that yield potential does not determine optimal input application. This research proposes a microeconomic theory based approach to selecting variables for nitrogen management zone delineation, using a spatial error model for yield estimation and Moran's I plots for delineating management zones. Although nitrogen management is the focus of this research, the process can be applied to other agricultural inputs.

Microeconomic theory makes clear that economically optimal input application strategies do not necessarily depend on yield levels; rather, they depend on the responses of yields to inputs. The response of yield to many different factors, including managed inputs and soil and field characteristics (e.g., electroconductivity and slope) is known as a yield function, and the marginal product of nitrogen is the derivative of this function with respect to nitrogen. Solving the profit-maximization problem with this yield function, the optimal nitrogen rate for a plot is given by a function of the variables in the marginal product of nitrogen. The optimal nitrogen equation demonstrates that using "yield zones" to determine "management zones" is likely to be a suboptimal strategy, and these zones should instead be delineated using characteristics that affect the yields' response to inputs. Specifically, a management zone should be an area of the field with the same marginal product function with respect to the input being managed.

Note that typical profit-maximization does not consider the cost of data collection which does not appear in the production function but, depending on the type, can be quite expensive. For instance, deep soil sampling to measure organic matter and nitrogen in the soil is labor intensive making the cost higher than many other data. To inform the choice to collect soil characteristics, the value of these data should be evaluated in terms of the profit gained from their collection. Thus, the yield function and the coefficient on the interaction between the input and the field characteristic of interest are needed to determine the potential profits gained from data collection.

This research contributes to a large literature on management zone delineation for variable rate application of inputs. Early work used organic matter, phosphorous, and insoil nitrogen collected in grid sampling or soil maps to produce zones (Carr et al. 1991, Ferguson et al. 1996, Mausbach et al. 1993). Due to the cost of soil sampling, this research also tried to determine the optimal density of sampling for accurate management zones while minimizing costs, and Ferguson et al. found even low density soil maps

produced higher profits than a single-rate application (1996.) Conversely, homogenous Midwest fields may not benefit from variable rate application of inputs as noted by Bullock et al. 1998.

As a lower cost alternative to soil sampling, yield maps were also considered for the delineation of management zones. Because weather causes yield to vary from year to year, multiple years of yield maps are recommended for any management zone delineation (Ferguson et al. 1996, Horbe etal. 2013, King 2005). The maps are used to identify consistently low or high yield areas of the field, as well as, parts of the field with inconsistent yields. Management zones created using yield maps have shown increased profits, as seen in a 2013 study on seed rate (Horbe et al. 2013). Rather than for defining management zones, another piece research identifies yield maps as a way to assess field variability and, thus, suitability for variable rate application (King 2005).

Another variable used in the delineation of management zones is electroconductivity (EC) which measures the ability of soil to conduct electrical current. The use of EC in zoning does not require multiple years of data, but past research has shown both positive and negative relationships between the variable and yield due to the underlying soil characteristics that EC values represent (Kitchen et al. 2001, Kravchenko et al. 2002). For example, EC has been shown to be correlated with water, salt, silt, clay, and sand content, drainage, and organic matter in the soil which are all variables affecting the productivity of nitrogen application (King 2005; Kweon et al. 2013). Kravenchenko et al. 2003 found that EC had a negative effect on yield when there was high March precipitation; this result is consistent with high EC values in Illinois being associated with high levels of clay, water content, and poor drainage. Soil type can also explain the relationship between EC and yield because soils contain different types of salts with different relationships to crop yield (King 2005). Thus, current research generally includes EC along with several other variables, such as elevation, slope, and soil type, when defining management zones (King 2005, Kweon et al. 2012, Velandia et al. 2008).

Recent research uses three steps: identifying variables associated with yield, choosing the number of zones, and then using cluster analysis to define management zones. Each step of this process can be accomplished through multiple methods. Principal component analysis is the most common way to choose the relevant variables (Gustaferro et al. 2010, King 2005, Peralta et al. 2013, Tagarkis et al. 2013, Yan et al. 2007). Normalized classification entropy has been the dominant way to determine the optimal number of management zones through balancing the variation within a zone and the variation across zones, but alternative methods have been proposed by Zhang et al. in 2010 and Vendrusculo and Kaleita in 2011. Similarly, fuzzy c-means clustering has been the most common way to delineate management zones with the chosen numbers of zones and characteristics variables, but Velandia et al. proposed a new method to account for spatial correlation in 2008. By using Moran's I scatter plots, these zones account for the spatial structure of the field or soil characteristics. There are three resulting zones for any variable, high-high, low-low, and mixed. The high-high and low-low zones are comprised of the plots with similar neighbors while the mixed zones are plots with dissimilar neighbors.

Another limitation of the past literature is the lack of zone prescriptions and economic analysis; many of the studies mentioned establish management zones without determining the input rates for each zone or evaluating the profitability of the zones compared to the optimal uniform input rate for a field. Rather, these studies tend to measure the validity of a management zone based on the variation of the characteristic within and across zones. This research will estimate a quadratic yield function using a spatial error model. Soil and field characteristics will be chosen from the nitrogen interaction terms in the yield estimation, and the zones will be defined with the Moran's I cluster analysis proposed by Velandia et al. in 2008. To compare the management zones to a uniform rate, I will simulate the yields from both approaches using the APSIM model which incorporates many models of the growing process into a predictive algorithm (Keating, B. A., P.S Carberry, G.L Hammer et al. 2003). Although microeconomic theory suggests the proposed management zones will result in higher profits, this difference may not be statistically significant.

The results of this research inform both future literature and commercial activity in precision agriculture. Currently producers have machinery that enables them to apply variable rate inputs, and they collect data about the field to establish those rates. Further, the methods used to collect soil characteristics are labor intensive, resulting in high costs for producers and collection of this data every few years rather than annually. With data containing both EC and soil properties from a soil test, this research also shows the value in terms of economic profits of collecting soil samples rather than using EC as a proxy for soil properties. The common methods to establish management zones rely on the concept of yield-limiting factors, and much of literature does not discuss how to determine optimal rates after zoning. With the establishment of theoretically consistent new methods for delineating management zones, management advice may improve, and the value of data collection such as soil sampling and electroconductivity measurement will be better understood. If traditional soil tests are not necessary for profitable management zone delineation, producers would benefit from eliminating these activities.

#### Data

These data come from a 2017 completely randomized agronomic nitrogen and seed rate trial on a 113-acre farm growing corn in Central Illinois. The field data collected are electroconductivity, elevation, nitrogen applied, seed rate applied, and dry yield per acre. Elevation was transformed into slope and aspect, which are also considered in the estimation procedures. Additionally, the producer supplied a recent soil test containing the analysis of 46 samples. Soil properties included are organic matter, calcium content, phosphorous, cation exchange capacity (a measure of the soil's ability to hold essential nutrients), potassium, and other contents not considered in this research. To estimate the soil characteristics in a subplot, inverse distance weighting interpolation was used on a 114 by 150 foot grid. Equation 1 represents the interpolated value of the characteristic *c* at any point *x*, where  $w_i(x)$  is a weight assigned to sample point  $x_i$  based on the distance between *x* and  $x_i$ .

$$c(\mathbf{x}) = \sum_{i=1}^{46} w_i(\mathbf{x}) c(\mathbf{x}_i)$$
(1)  
$$w_i(\mathbf{x}) = \frac{d(\mathbf{x}, \mathbf{x}_i)^{-1}}{\sum_{i=1}^{46} d(\mathbf{x}, \mathbf{x}_i)^{-1}}$$
(2)

The results of the interpolation for calcium and the original sampled points are shown in Figure 1; note that only the right side of the figure is the part of the field this study is analyzing.

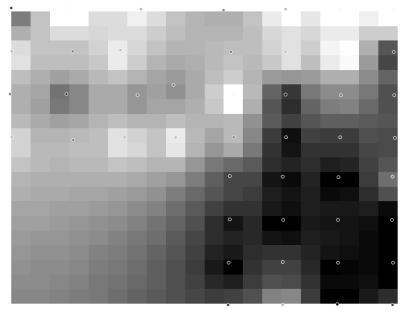


Figure 3. Results of Interpolation for Calcium

Experimental plot size was determined by the width of the machine's applicators and the known errors of yield monitors; on this field the plots were 60 feet wide and 280 feet long. The producer's sprayer had two 60 feet wide sections, a 30 feet wide planter, and a 60 feet wide yield monitor. Thus, a width of 60 feet gives accurate application of inputs and retrieval of trial yield. Additionally, monitors do not respond immediately to large changes in yield, such as when the machine moves from a low nitrogen treatment to a high nitrogen treatment. For this reason, plots were designed to be long enough to eliminate observations on the width-end of a plot where errors in the yield monitor are more likely to occur while maintaining an adequate number of interior observations.

The exact length needed for accurate yield is not known and depends on the change in yield and the yield monitor used. The length of the 280 feet was considered the best length given the constraints mentioned. To clean out unreliable yield observations, 30 feet was considered a conservative amount to eliminate from each plot. One way to assess this cleaning process is to compare the variability of yield before and after cleaning the plots. The average yield was around 230 pounds per acre in both datasets, but the

standard deviation reduces to 15 from 42 when the ends of plots are removed. Thus, the cleaning reduced the noise in the yield variable but did not alter the mean value of yield on the field.

Plots on the edges of the field and partial plots are also eliminated because there are too few observations in small plots and the driving patterns on the edge of the field result in unreliable data. As seen in Figure 2, there are partial plots on the bottom and top of the field which are eliminated from the data for analysis as this is the area where the machinery turns around, leading to errors in the measured yield or applied input amounts. There were 237 plots in this field, and after removing the 30 feet off of the ends, each plot was divided into 4 subplots in which the median of these data was calculated, resulting in 948 observations. These subplots can be seen in Figure 3, where 77 partial and edge plots are deleted, leaving 640 subplots for analysis. The nitrogen treatments were 0, 7, 13 and 20 gallons of nitrogen per acre, and seed rates were 27, 31, 34, and 37 thousand plants per acre. The rates the producer would apply without participation in the trial were 13 gallons of 28% nitrogen and 37 thousand plants per acre. The producer applies a base level of nitrogen at planting of 160 pounds of nitrogen, so the applied rates here are additional pounds of nitrogen. The resulting treatments are 160, 180, 200, and 220 pounds of nitrogen per acre.

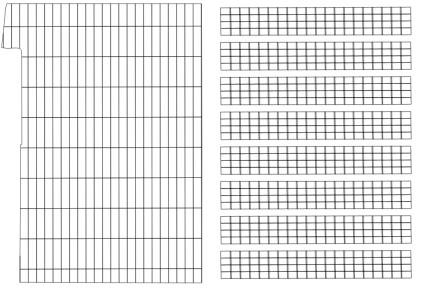


Figure 1. Plot Design

Figure 2. Subplot Design

## **Descriptive Statistics**

Like yield monitors, but to a lesser extent, input applicators also have a lagged response to changes in experimental rates, so there is reason to compare the applied amounts to the designed amounts. Table 1 indicates the applicator did not consistently apply the nitrogen rates, particularly the zero rate. While 160 of the subplots were designed to have zero nitrogen applied, only 97 of the plots had zero nitrogen applied. This is likely the result of a zero nitrogen plot designed next to a higher rate plot, where the machine does

not apply the correct amount immediately. Table 2 shows that the seed applicator was accurate over the whole range of seed rates designed. Overall, the accuracy of the applicators resulted in a smaller range of nitrogen rates in the data, which limits the ability to predict yield at low levels of nitrogen.

Table 1. Accuracy of Nitrogen Applicator						
Designed N-	Average Applied	Standard Deviation of				
Rate	N-Rate	Applied N-Rate				
0	6.22	8.30				
7	10.35	3.87				
13	15.44	2.73				
20	19.18	4.03				

## Table 2. Accuracy of Seed Applicator

Designed S-	Average Applied	Standard Deviation of				
Rate	S-Rate	Applied S-Rate				
27	27.06	0.10				
31	31.03	0.08				
34	33.99	0.08				
37	36.96	0.12				

Previous research has shown that variable rate application of inputs results in higher profits for fields with larger variation in field and soil characteristics (Bullock et al. 1998, Thrikwala et al. 1999). Additionally, the homogeneity of fields may also result in fewer management zones. In Table 3 I present the mean and standard deviation of the potential covariates in the yield equation; these statistics suggests there is little variation in elevation or slope to create management zones. However, there is variation in EC and several soil properties, suggesting these variables could be used to define management zones.

Table 5. Descriptive ofatisties of field and obvariates						
Variable	Mean	Standard Deviation				
yield	229.60	15.06				
slope	1.07	1.51				
elevation	627.67	3.04				
ec_shallow	29.51	7.22				
ec_deep	37.23	8.53				
om	2.41	0.07				
k	215.88	18.83				
р	69.62	8.99				
ca	2866.35	289.08				
cec	9.30	0.62				

#### Table 3. Descriptive Statistics of Yield and Covariates

If EC can substitute the more expensive method of soil sampling for determining management zones, it should be correlated with the soil properties. Table 4 shows that ec\_deep and ec\_shallow are correlated with all of the properties except potassium,

suggesting the measure could produce profitable management zones if those soil properties are affecting the marginal product of nitrogen. Elevation is also correlated with all soil properties, including potassium; however, its lack of variation makes it an ill-suited variable for the delineation of management zones. Table 5 shows the randomization of the seed and nitrogen rates resulted in the applied amounts being uncorrelated with the covariates as would be expected.

	ec_s	ec_d	elev	slope	от	k	р	са
ec_s	1							
ec_d	0.98***	1						
elevat	-0.04	-0.08	1					
slope	0.04	0.04	0.10**	1				
om <sup>.</sup>	0.12***	0.15***	-0.46***	-0.15***	1			
k	-0.04	0.00	-0.56***	-0.04	0.31***	1		
p	0.09**	0.13***	-0.40***	-0.07*	0.29***	0.62***	1	
са	-0.12***	-0.09**	-0.66***	-0.20***	0.40***	0.56***	0.50***	1
cec	-0.17***	-0.15***	-0.51***	-0.21***	0.37***	0.54***	0.48***	0.92***

#### **Table 4. Correlation of Covariates**

#### Table 5. Correlation of Treatment Variables and Covariates

S	n
1	
-	1
0.03	
-	-0.03
0.05	
-	-0.04
0.05	
-	-0.06
0.01	
0.02	0.08
0.05	-0.04
-	0.05
0.01	
-	-0.03
0.00	
0.04	0.01
0.07	0.02
	1 - 0.03 - 0.05 - 0.05 - 0.01 0.02 0.05 - 0.01 - 0.01 - 0.00 0.04

### **Yield Estimation and Results**

There is a large existing literature regarding the functional form of yield's response to inputs. Popular forms include quadratic, quadratic plateau, linear plateau, von Leibig, and the Misterlich-Baule form. The last three are nonlinear forms that allow for

nonsubstitutability between inputs, which is consistent with Leibig's Law of the Minimum (Llewelyn and Featherstone 1996). The Law of the Minimum says that maximum yield is related to the most limiting growing factor; thus, additional units of any other input will not increase yield. Studies comparing functional forms have found the quadratic form overestimates the maximum yield, resulting in higher optimal nitrogen rates (Llewelyn and Featherstone 1996). In the future, estimation will use a quadratic plateau with spatial errors. For the purposes of this paper, the quadratic form, despite this bias, will be used. The resulting optimal nitrogen rates will be biased for both delineation techniques, but the comparison should still be valid.

The functional forms above treat nitrogen as a continuous variable, but the designed trials can lead to categorical rather than continuous input variables. The nature of these variables is why the crop science literature often uses analysis of variance for analyzing the results of trials, but those trials use manual application of inputs ensuring accurate rates. As Tables 1 and 2 demonstrate, using the precision technology does not result in the same accuracy. Based on the results of those tables and the plots in Figures 4 and 5, seed should be treated as a categorical variable in the yield estimation while nitrogen can be considered continuous.

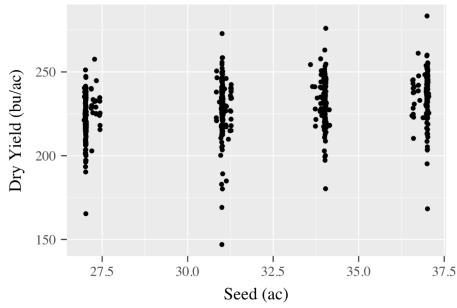


Figure 4. Plot of Seed Against Dry Yield to Demonstrate Categorical Nature of Seed Variable

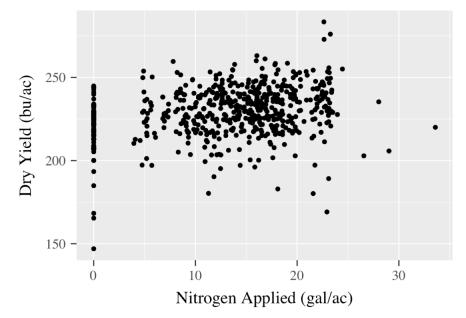


Figure 5. Plot of Nitrogen Against Dry Yield to Demonstrate Continuity of Nitrogen Variable

An ordinary least squares (OLS) estimation of the yield function is likely to exhibit spatial correlation in the residuals because there are unobserved variables affecting yield that are spatially correlated; this violation of spherical errors in the OLS model results in a loss of efficiency for the OLS estimates. Three models were compared to investigate the spatial correlation in the yield model: OLS, a spatial lag, and a spatial error.

The neighborhood structure used in the analysis is a Queen structure as presented in Figure 6, where the surrounding yellow subplots are considered neighbors of subplot *a*. The alternative rook neighborhood structure was also considered for the models with negligible differences in the results. The first spatial model is a spatial lag estimation that allows for the observed dependent variable to be spatially correlated; the second is a spatial error estimation that allows the residuals to be spatially correlated. An example of when the spatial lag model is used is the estimation of flu incidence, where an individual is more likely to have the flu if his neighbors have the flu. As the yield of a subplot is unlikely to affect the yield of a neighboring plot, it is more likely that the underlying soil and land characteristics are responsible for spatial correlation in the observed yield. Thus, the spatial error model is expected to be the best procedure for eliminating the spatial correlation in the error term.

		а					

Figure 4. Queen Neighborhood Structure for Spatial Analysis

Using the two specifications presented in Table 6, the OLS and spatial lag models had spatially correlated errors according to the Moran's I estimate on the residuals while the spatial error model had an insignificant Moran's I. Thus, the spatial error model is the preferred estimation procedure. Equation 3 shows the specification of this model, where X contains the variables in the restricted and unrestricted model and  $\lambda$  is the estimate of spatial correlation of the error term.

$$y = X\beta + \lambda W u + \varepsilon \tag{3}$$

The results in Table 6 suggest there is a difference in the yield estimation when including the results of the soil test. When only topography and EC are available, nitrogen interacts with elevation which is highly correlated with the soil contents as seen in Table 4. The negative sign on this interaction indicates higher elevation decreases the marginal product of nitrogen. EC only affects nitrogen linearly with EC decreasing yield. Nitrogen and seeding rates are significant and have the expected signs.

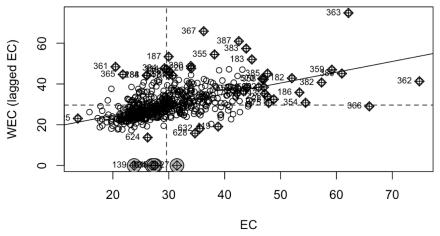
The full model results in Table 6 suggest calcium and EC are the characteristics for the delineation of management zones. Although the results are consistent with the restricted model in terms of the seeding rate coefficients, the nitrogen and nitrogen interactions are notably different in the full model. Once organic matter is added in the specification, nitrogen applied is insignificant, and the interaction between EC and the squared nitrogen term is significant while the squared nitrogen coefficient is insignificant. The positive sign on the nitrogen and calcium interaction terms indicates high levels of calcium increase the marginal product of nitrogen, a result that is consistent with the Law of the Minimum. The EC and nitrogen interaction suggests that increased levels of EC result in a faster decrease in the marginal product of nitrogen.

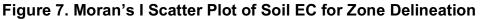
	Restricted Model	Full Model
Intercept	227.814***	145.130***
	(3.097)	(22.864)
п	23.790***	-0.0513
	(8.974)	(0.482)
$n^2$	-0.0104***	х ,
	(0.006)	
<i>s</i> =31	6.924 <sup>***</sup>	6.293***
	(1.876)	(1.797)
<i>s</i> = 34	9.913 <sup>***</sup>	9.6921 <sup>***</sup>
	(1.904)	(1.822)
<i>s</i> = 37	14.265***	13.790***
	(1.949)	(1.869)
ec_d	-0.386***	-0.332***
	(0.065)	(0.075)
n * e	-0.0366***	(0.010)
	(0.014)	
om	(0.011)	33.563***
011		(9.619)
n * ca		0.0003***
n + cu		(0.0001)
$n^2 * ec$		-0.0004***
		(0.0002)
λ	0.292	0.271
Moran's I on		
Residuals	0.378***	0.230***
Residuais		

Table 6. Results for the Full and Restricted Covariates

Given the spatial correlation in the data, Velandia et al.'s approach to management zone delineation seems appropriate for this research. Thus, Figure 7 shows the Moran scatter plot as proposed by Anselin 1996 for EC. The slope of the line is the Moran's I statistic that represents positive correlation in EC; that positive correlation can be seen in the plot with most points falling in the upper-left (high EC values around other high EC values) or lower-right corners (low EC values around low EC values) of the plot. The next step is to take this plot and create a map of the field with the three types of subplots (high, low, and mixed). With zones established, the optimal nitrogen rates will then be assigned and evaluated. This evaluation will compare the restricted and full model to each other as well as to the yield based management zones that have not been established yet.

These preliminary results suggest there is information contained in the soil test that can be used for management zone delineation, but the evaluation of the resulting management zones is necessary to determine the real benefits from collected these data. Additionally, the stability of the selected soil characteristics will change the amount of time management zones can be considered reliable from a given soil test. The final results will consider this aspect with discounted future benefits.





## References

Bullock D. G., D. S. Bullock, E. D. Nafziger, T. A. Doerge, S. R. Paszkiewicz, P. R. Carter, & T. A. Peterson. (1998). Does Variable Rate Seeding of Corn Pay? *Agronomy Journal*, 90:830-836.

Carr, P.M., G. R. Carlson, J. S. Jacobsen, G. A. Nielsen, & E. O. Skogley. (1991). Farming Soils, not fields: A strategy for increasing fertilizer profitability. *Journal of Production Agriculture*, 4:57-61.

Ferguson, R. B., C. A. Gotway, G. W. Hergert, & T. A. Peterson. 1996. Soil Sampling for Site- Specific Nitrogen Management. *Precision Agriculture*.

Gustaferro, F., A. Castrignano, D. Debendetto, D. Sollitto, A. Troccoli, & B. Cafarelli. (2010). A comparison of different algorithms for the delineation of management zones. *Precision Agriculture*, 11:600-620.

Horbe, T. A. N., T. J. C. Amado, A. O. Ferreira, & P. J. Alba. (2013). Optimization of corn plant population according to management zones in Southern Brazil. *Precision Agriculture*, 14:540-465.

Keating, B. A., P.S Carberry, G.L Hammer et al. (2003). An overview of APSIM, a model designed for farming systems simulation. *European Journal of Agronomy*, 18:267-288

King, J. A., P. M. R. Dampney, R. M. Lark, H. C. Wheeler, R. I. Bradley, & T. R. Mayr. (2005). Mapping Potential Crop Management Zones within Fields: Use of Yield-map Series and Patterns of Soil Physical Properties Identified by Electromagnetic Induction Sensing. *Precision Agriculture*, 6:167-181.
Kitchen, N. R., S. T. Drummond, E. D. Lund, K. A. Sudduth, & G. W. Buchleiter. (2003).

Soil

Electrical Conductivity and Topography Related to Yield for Three Contrasting Soil–

Crop Systems. Agronomy Journal, 95:483-495.

Kravchenko, A. N., K. D. Thelen, D. G. Bullock, & N. R. Miller. (2003). Relationship among Crop Grain Yield, Topography, and Soil Electrical Conductivity Studied with Cross- Correlograms. *Agronomy Journal*, 95:1132-1139 Kweon, G. (2012). Delineation of site-specific productivity zones using soil properties and topographic attributes with a fuzzy logic system. *Biosystems Engineering*, 112:261-277.

Llewelyn, R. V. & A.M. Featherstone. (1996). A Comparison of Crop Production Functions Using Simulated Data for Irrigated Corn in Western Kansas. *Agricultural Systems,* 54:521-538.

Mausbach, M. J., D. J. Lytle, & L. D. Spivey. (1993). Application of Soil Survey Information to Soil Specific Farming. *Soil Specific Crop Management*.

Peralta, N. R., J. L. Costa, M. Balzarini, & H. Angelini. (2013). Delineation of zones with measurement of soil apparent electrical conductivity in pampas. *Canadian Journal of Soil Science*, 93:205-218.

Tagarkis, V. Liakos, S. Fountas, S. Koundouras, & T. A. Gemtos. (2013). Management zones delineation using fuzzy clustering techniques in grapevines. *Precision Agriculture*, 14:18-39.

Thrikwala, S. A. Weersink, G. Kachanoski, & G. Fox. (1999). Economic Feasibility of Variable-Rate Technology for Nitrogen on Corn. *American Journal of Agricultural Economics*, 81:914-927.

Velandia, M., R. M. Rejesus, K. Bronson, & E. Segarra. (2008). Economics of Management Zone Delineation in Cotton Precision Agriculture. *Journal of Cotton Science*, 12:210-227.

 Vendrusculo, L. G. & A. L. Kaleita. "Modeling zone management in precision agriculture through Fuzzy C-Means technique at spatial database." A Presentation at the ASABE Meeting, Louisville, KY, August 2011.

Yan, L., Z. Shi, C. Wu, H. Li, & F. Li. (2007). Determination of potential management zones from soil electrical conductivity, yield and crop data. *Journal of Zhejiang University*, 9:68-76.

Zhang, X., L. Shi, X. Jia, G. Seielstad, & C. Helgason. (2010). Zone mapping application for precision-farming: a decision support tool for variable rate application. *Precision Agriculture*, 11:103-114.