Development of an Online Decision-Support Infrastructure for Optimized Fertilizer Management

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A paper from the Proceedings of the
14th International Conference on Precision Agriculture
June 24 – June 27, 2018
Montreal, Quebec, Canada

Abstract. *Determination of an optimum fertilizer application rate involves various influential factors, such as past management, soil characteristics, weather, commodity prices, cost of input materials and risk preference. Spatial and temporal variations in these factors constitute sources of uncertainties in selecting the most profitable application rate. Therefore, a decision support system (DSS) that could help to minimize production risks in the context of uncertain crop performance is needed. This paper presents a newly developed framework for a dynamic DSS, called NumericAg, which seeks to estimate the probability of achieving expected profits under specific growing conditions. The proposed system includes a database, a user interface, and a numeric engine for computation of profit space. The online web interface (www.numericag.com) allows a user to specify production conditions (e.g., previous crop, tillage system, soil type, organic matter content, rainfall, and crop heat unit), and to accept or modify the price of grains and fertilizers. The database stores the results of previously recorded fertility trials. The profit space computation engine was designed to estimate the probability of achieving different levels of net return over the cost of nitrogen fertilizer for every potential application rate. The profit space engine considers over 20,000 potential quadratic-plateau fertilizer response functions in combination with 49 cost scenarios evaluated against every fertility trial weighted according to the growing conditions specified by the user. Consequently, probability of different levels of potential net return over cost of fertilizer was estimated using fertilizer response observations that relatively closely match the growing conditions specified by the user. A sensitivity analysis was used to show DSS response to changes in specified growing conditions. Thus, when comparing different growing conditions, a smaller probability of achieving relatively high profits was found with sandy soils, relatively low crop heat units and water availability, or low levels of nitrogen contribution* *from previous crops. Although the rate that maximized the expected net return over cost of nitrogen did not change substantially, the rate of profit decline due to under application of nitrogen fertilizer was different for different growing conditions.*

Keywords. *decision support system, numeric analysis, nitrogen fertilization, corn, profitability.*

Introduction

Crop response to a specific fertilizer input can be influenced by a number of undocumented or unknown factors that cause uncertainties. The uncertainty of spatial and temporal information can be a major impediment to good decision outcomes (McBratney *et al*., 1997). Producers like to achieve maximum returns from the fertilizer application and often over-apply nitrogen fertilizer to corn because of the uncertainty in predicting the economic optimum nitrogen rate (*EONR*) (Dellinger *et al*., 2008). Also, the optimal rates of nitrogen for corn are difficult to determine because they depend mainly on the interactions between weather, soil and crop management factors (Tremblay *et al*., 2010). The over-application of fertilizers leads to soil degradation and increased costs, whereas application below a sufficient rate incurs profit losses (Bullock & Bullock, 1994; Assimakopoulos *et al*., 2003). Nitrogen (N) recommendation rates provided by agronomists and soil and fertilizer consultants vary by soil and by crop across Canada (Yang *et al*., 2006). Thus site-specific growing conditions should be considered while estimating the optimum application rate.

This paper presents an extension to the initial framework proposed by Adamchuk *et al*. (2017). The proposed DSS dynamically links the site-specific spatial, temporal and management factors, in the context of uncertain information (Bachmaier, 2012), to production and profit functions, which enables the estimation of the *EONR* to maximize net return over the cost of fertilization (NRCF) or to increase the probability of achievement of certain levels of NRCF. Production uncertainty was modeled through the probability of the residuals between observed and predicted yields (estimation error), and economic uncertainty was modeled from the probability of different crop prices and costs of fertilizer. The uncertainty-based treatment of each model input allows for a balance between the potential results of under-application or over-application. To illustrate this framework, an online prototype DSS was developed to optimize nitrogen fertilization for corn using fertility trial data assembled through research experiments across Central Canada.

Materials and Methods

The proposed DSS consists of several key components (Figure 1): 1) a user interface, 2) a database, 3) access to public online resources, and 4) a numeric engine for user interaction, computation of profit space and result visualization.

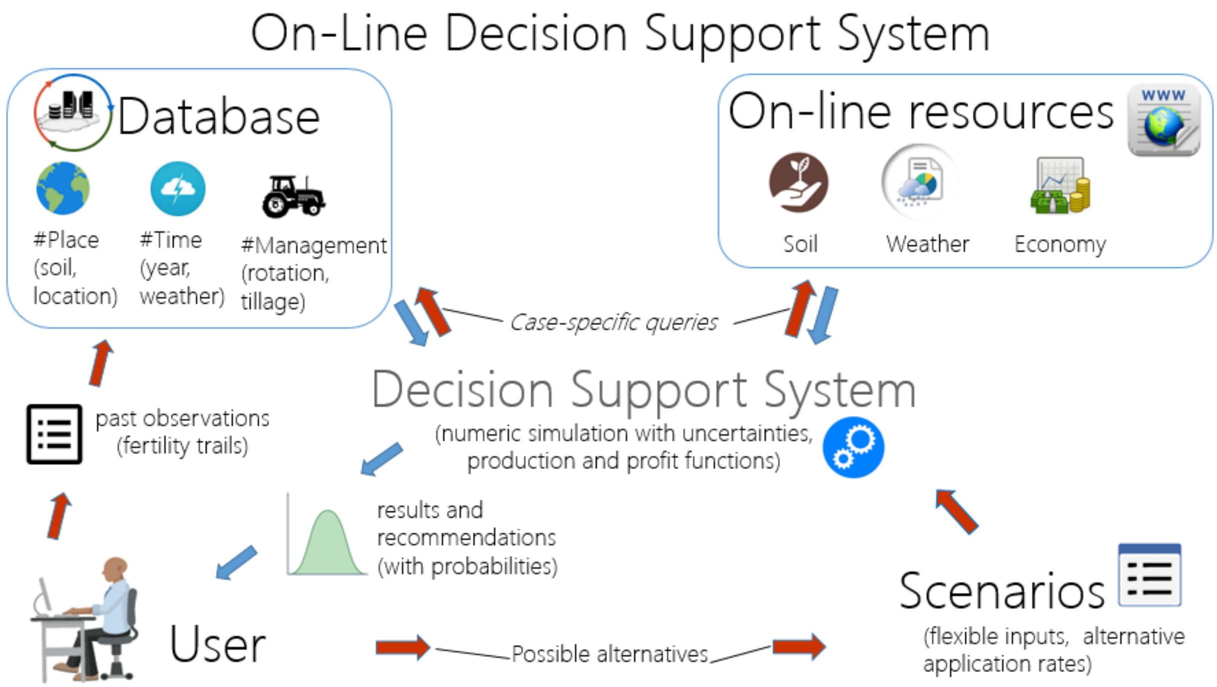

Figure 1. Architecture of the proposed decision support system.

Through a mobile interface, available online at *www.numericag.com*, the user specifies his/her crop growth conditions and practices, such as previous crop, tillage system, soil type, organic matter content, rainfall, and crop heat unit. Also, the user has the ability to modify the pre-populated prices of grains and fertilizers, retrieved in real-time from the Chicago Mercantile Exchange (CME). These constitute the basic inputs used to construct alternative scenarios. Complementary input values, such as soil information, weather and prices, could eventually be retrieved from alternative online sources. The initial database, currently implemented in MySQL, includes 320 records of yield data replicated for 5 N fertilization rates (0 to 200 interpolated to increments of 50 kg/ha). This means a total of 1680 fertility trials, resulting from an extensive meta-analysis study (Tremblay *et al*., 2012). Each record is linked to a site and a year. It includes a corn yield estimate and the corresponding nitrogen application rate, as well as weather data, soil conditions and management practices. At the same time, previous observations pertaining to completed fertility trials can be entered in the database when available. That way the DSS can integrate the newly entered data to adapt and generate up-to-date output.

Profit (objective function) was defined as the net return over cost of fertilizer (NRCF):

$$NRCF = Y \cdot c_Y - N \cdot c_N \tag{1}$$

where Y is the crop yield predicted as a function of the N fertilization rate (t/ha), c_Y is the price of the harvested crop (\$/t), N is the specified fertilizer application rate (kg/ha) and c_N is the cost of fertilizer (\$/kg). For any specified N rates, it is possible to calculate *NRCF* values and their probability of occurrence for different combinations of possible values of Y, c_Y and c_N.

The probability of obtaining a specific *NRCF* was calculated by the joint probability of occurrence of each of its component, assuming they are independent of each other:

$$p(NRCF) = p(Y) \cdot p(c_Y) \cdot p(c_N) \tag{2}$$

where $p(Y)$ is the probability of yield Y, $p(c_Y)$ is the probability of yield price c_Y; and $p(c_N)$ is the probability of fertilizer cost c_N. The goal of the DSS is to calculate the whole range of possible outcomes and their associated probabilities of occurrence, for all possible values of the production factors under control.

The heart of the DSS is comprised of a record similarity assessment mechanism that weighs database trial data with respect to the production context (growing conditions) specified by the user. The production context for each record may correspond very well, only partially or not at all to the context defined by a DSS user. Our proposed approach to handle partial correspondence to the user context was based on the calculation of a similarity index (λ) for each record present in the dataset: records with higher similarity would play a more significant role in model assessment and vice versa. In the current version of the prototype, non missing feature records from the database were retrieved (1,140 records) with SQL statements and a similarity index was calculated for each record using such features as soil type (e.g., sandy, clay, silt), precipitation (e.g., dry, medium, wet) based on the concept of abundant and well distributed rainfall (AWDR) proposed by Tremblay *et al*., (2012), temperature (e.g., cold, warm, hot) based on the crop heat unit (CHU) described in Bootsma *et al*., (2005), as well as tillage practices (i.e., till or no-till) and N contribution of preceding cultivar (e.g., weak, medium, strong). Each similarity feature (k) was discretized and provided as options for the user to ease the specification of the conditions prevailing on his/her site. The system then transforms the feature's category into its continuous numerical value equivalence, to be used in the similarity assessment function.

Record similarity assessment

Several criteria were established for the development of the record similarity assessment mechanism. First, it should consider the fact that some features are more important than others when assessing similarity. Second, it should be highly sensitive to the presence or absence of a match, especially for essential features. Thirdly, it should be able to handle numeric, continuous features. The approach that was adopted was based on a product, which rapidly decreases the similarity between records towards its minimum value in the absence of a match for at least one feature, which in turn decreases their impact when fitting models. A power value was used to decrease further the similarity index if the database record does not match with the user context. To simplify the presentation, let's assume that the record representing the user context is *u*. The similarity $\lambda_{i,u}$ between any j^{th} record (a database trial) and the user specified record *u* can be calculated based on all *k* features as:

$$\lambda_{j,u} = \prod_{k=1}^{K} \left(1 - \delta\lambda_k \left| \frac{x_{k,j} - x_{k,u}}{x_{k,\max} - x_{k,\min}} \right| \right)^q \tag{3}$$

where $\delta\lambda_k$ is the weight (between 0 to 1) affiliated with the k^{th} feature, $x_{k,j}$ is the value of the k^{th} feature of record *j*, $x_{k,u}$ is the value of the k^{th} feature of user context *u*, $x_{k,max}$ and $x_{k,min}$ are the maximum and minimum values found in all records for the k^{th} feature, and q is the power of similarity. $x_{k,max}$ and $x_{k,min}$ are used to standardize values within a range from 0 to 1. Increasing the value of q reduces the influence of records that do not match well user inputs.

Another factor discussed earlier is the importance of the feature. Some features may have greater influence then others on yield responses (based on metadata analysis), and a higher weight value should be associated with these features when calculating the similarity index. For example, the soil type has greater influence on the yield response than the tillage system. In the current version, the weights for all features were assigned based on domain knowledge and preliminary analysis, but they could be changed in future. When a user defines a context (growing conditions), the DSS calculates a similarity index (λ) for each record in the database with values ranging from 0 to 1, and where values closer to 1 indicate higher resemblance.

Production function

A quadratic-plateau (QP) equation was used as the production function to model and predict corn yield in response to N fertilization (Figure 2). The QP model is known to be a good fit with biological response (Bullock and Bullock, 1994; Bongiovanni and Lowenberg-DeBoer, 2000; Adamchuck, 2013). Thus, yield response to nitrogen fertilization was defined as:

$$Y = \begin{cases} a_0 + a_1 N + a_2 N^2 & for \quad N < N_{Y\max} \\ a_0 + a_1 N_{Y\max} + a_2 N_{Y\max}^2 & for \quad N \geq N_{Y\max} \end{cases} \tag{4}$$

where Y is the crop yield (t/ha), N is the specified fertilizer application rate (kg/ha), and N_{Ymax} is the minimum fertilizer application rate resulting in the maximum yield. The a_0, a_1 and a_2 are the coefficients of a second-order polynomial representation of yield response to N application rates below N_{Ymax}.

As shown in Figure 2, the parameters of Equation 4 can be defined through physical parameters: Y_0 (yield with no fertilization), Y_{max} (maximum achievable yield) and N_{Ymax} (mimimum N rate for maximum yield). This way, the coefficients of the second-order polynomial can be rewritten as:

$$a_0 = Y_0 \tag{5}$$

$$a_1 = \frac{2\left(Y_{\max} - Y_0\right)}{N_{Y\max}} \tag{6}$$

$$a_2 = \frac{Y_0 - Y_{\max}}{N_{Y\max}^2} \tag{7}$$

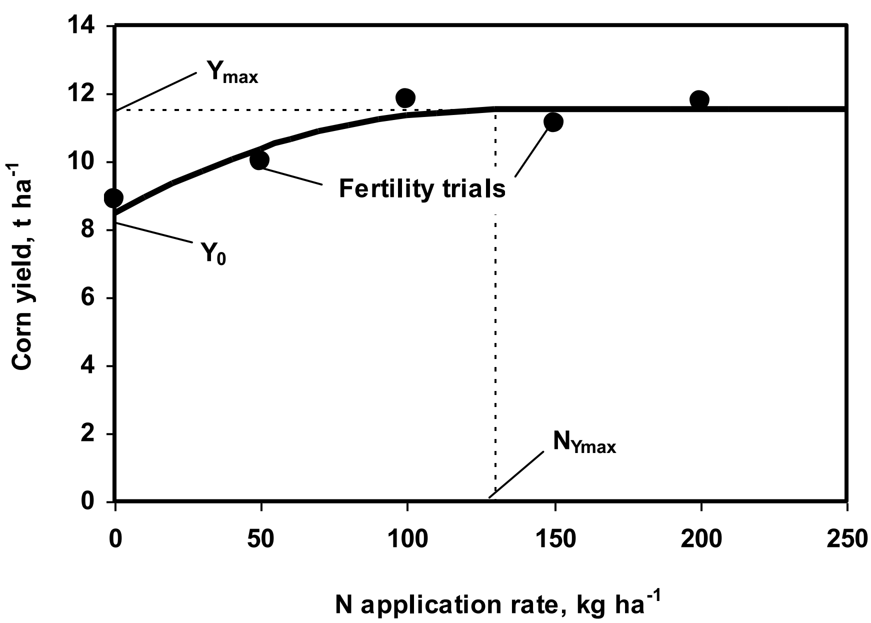

Figure 2. Example of a quadratic-plateau yield response model.

The traditional approach in a modeling process is to find the set of parameters that minimize errors or residuals. However, Bachmaier (2012) argued that the analysis of residuals alone may not be sufficient to assess the modeling results for N fertilizer studies. The main reason is that this does not assess accurately the reliability of the N_{Ymax} values derived from the quadratic models. Cerato and Blackmer (1990) previously showed that two models with very close R^2 values could lead to very different N_{Ymax}. This is valid also when comparing different sets of parameters for the same model. The approach that is proposed here is to consider all possible sets of model parameters within a physically plausible range and calculate the probability that each set of parameters results in no errors (i.e., probability that this model is the proper fit). Since $Y = f(N)$, $p(Y)$ reflects the accuracy with which the model predicts yield as a function of the N rate.

In the DSS, all possible sets of model parameters within a plausible range were considered for each N rate within a range of application rates. Values ranged from 0 to 19.5 t/ha with 0.5 t/ha increment for Y_0, from Y_0 to 20 t/ha with 0.5 t/ha increment for Y_{max}, and from 0 to 250 kg/ha with 10 kg/ha increment for N_{ymax}. The combination of these parameters (Y_0, Y_{max}, N_{ymax}) generates 21,525 possible production models for a given application rate (N).

Production probability assessment

The calculation of the probability that the yield predicted by any given model (or combination of production function parameters) be right, was based on the assumption that the errors of estimation for all trials in the database were normally distributed. For any model, the estimation error or residual was calculated through the difference between predicted yield and actual yield of a trial (database record), weighted by the record similarity index (λ):

$$\varepsilon_{\mathrm{i,j}} = \left(Y_i - Y_j\right) \cdot \lambda_j \tag{8}$$

where Y_i is the yield estimate (t/ha) for the i^{th} model (Equation 4), Y_j is the actual yield (t/ha) for the j^{th} trial from the database, and λ_j indicates the similarity of the j^{th} trial to the user's conditions (Equation 3). Therefore, the $\varepsilon_{i,j}$ is the yield estimation error for a combination of model i and database trial j.

Figure 3 illustrates the distribution of errors obtained for one of many possible QP models (i.e., a given combination of Y_0, Y_{max} and N_{Ymax}).

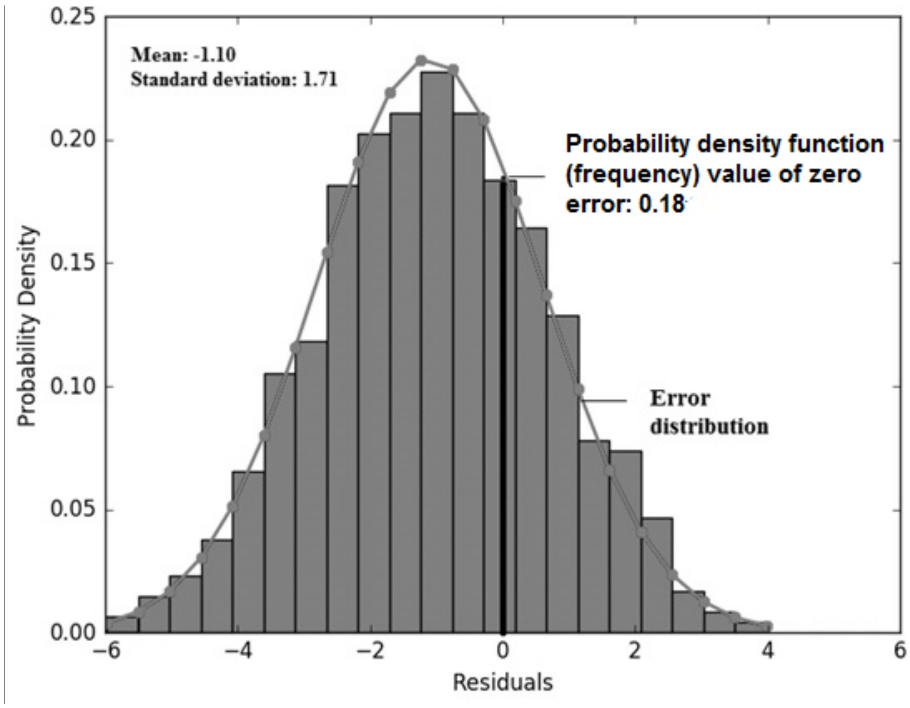

Figure 3. Example showing distribution of errors for an arbitrary model combination.

Estimation errors were used to obtain the frequency distribution value of zero error for a given production model using the normal probability density function (Evans *et al*., 2000):

$$freq(Y)_i = \frac{e^{\frac{(0-avg(\varepsilon_i))^2}{2*std(\varepsilon_i)^2}}}{\sqrt{2*\pi*std(\varepsilon_i)^2}} \tag{9}$$

where $freq(Y)_i$ is the probability density (frequency) of 0 error at the i^{th} model, calculated by providing average *avg* (ε_i) and standard deviation of errors *std* (ε_i). The average of weighted residuals was calculated as:

$$avg(\varepsilon)_i = \frac{\sum_{i=1}^{I} \varepsilon_i}{I} \tag{10}$$

where the *avg* $(\varepsilon)_i$ is the mean of the errors for the i^{th} model, calculated from the errors of all *j* records/trials (1,140 errors in total for each model). After obtaining the average, the standard deviation of weighted residuals can be calculated as:

$$std(\varepsilon)_i = \sqrt{\frac{\sum_{i=1}^{I} (\varepsilon_i)^2}{I-1}} \tag{11}$$

where *std* $(\varepsilon)_i$ was the standard deviation of the i^{th} model, using residuals calculated at all j^{th} records/trials. Additionally, the raw probability values were standardized so that the sum of $p(Y)_i = 1$:

$$p(Y)_i = \frac{freq(Y)_i}{\sum_{i=1}^{I} freq(Y)_i} \tag{12}$$

Finally, $p(Y)_i$ was the probability of achieving the yield Y_i estimated by model *i*, after normalization. The probability of getting and error of prediction of zero is proportional to the probability *p(Y)* for that specific model.

The probability of achieving yields at each possible combination of the model were calculated using the proposed errors to probability approach. However, the probability of achieving the estimated yield needs to be combined with the prices and costs probabilities as in equation 2 to integrate economic uncertainty associated with price/cost variations. Combining production and economic uncertainties will enable the generation of probabilities associated with possible profit margins for each *N* rate.

Computation of the profit space

The price (c_Y) and cost (c_N) were formulated in seven equal discrete values, from 130 to 220 ($/t) and from 0.4 to 1.6 ($ kg/ha), respectively. The probability of each discrete value was calculated using the normal distribution density function, with means and standard deviations provided in the user input form for price and cost, respectively. Figure 4 illustrates the distribution of prices and cost using given mean and standard deviation values.

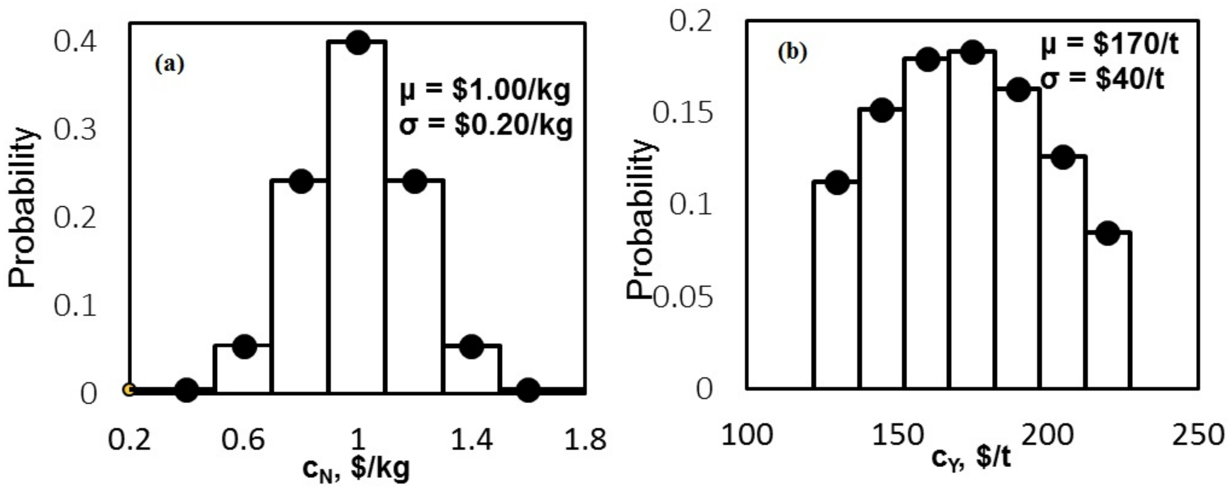

Figure 4. Distributions of the cost of fertilization (a) and price of yield (b) model inputs.

The *NRCF* for a specific case (i.e., for i^{th} model, l^{th} price and m^{th} cost) and any given *N* fertilization rate was calculated using:

$$(NRCF)_{ilm} = Y\left(Y_{0i}, Y_{\max i}, N_{Y\max i}, N\right) \cdot (c_Y)_l - N \cdot (c_N)_m \tag{13}$$

where Y ($Y_{0\,i}$, $Y_{max\,i}$, $N_{Ymax\,i}$, N) is the yield function derived by combining equations 1 and 4 together for i^{th} combination of production function parameters. The c_Y is the discrete price of the harvested crop ($/t) at index *l*, *N* is the nitrogen application rate (kg/ha) and $(c_N)_m$ is the discrete cost of fertilizer ($/kg) at index m. The probability of *NRCF* for a specific case (i.e., for i^{th} model, l^{th} price and m^{th} cost) was calculated using:

$$p(NRCF)_{ilm} = p\left(Y\right)_i \cdot p(c_Y)_l \cdot p(c_N)_m \tag{14}$$

where $p(Y)_i$ is the probability associated with the i^{th} model, which was itself calculated using the error to probability method. Likewise, $p(c_Y)_l$ is the probability of having a yield price c_Y at discrete value l, and $p(c_N)_m$ is the probability of the cost of fertilizer c_N being at an m^{th} discrete value.

Calculating all possible *NRCF* values and associated probabilities at each specific *N* rate produced the profit space at each application rate. With 49 combinations of possible costs of yield and fertilizer, combined with 21,525 production models, a total of 1,054,725 (21,525 x 49) *NRCF* values were calculated at each *N* rate, each with a corresponding *p(NRCF)*. These values can be summarized through the expected *NRCF* for each application N rate, which can be calculated for the whole range of possible rates, by joining Equations 13 and 14 together as:

$$NRCF(N) = \sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{m=1}^{M}\left((NRCF)_{ilm}.p(NRCF)_{ilm}\right) \quad (15)$$

where *NRCF(N)* represents the expected *NRCF* at the given application rate N, calculated by summing up the *NRCF* values multiplied by their respective *p(NRCF)*, for all combinations of models, prices and costs.

The expected fertilization benefit (*EFB*) can then be calculated, by comparing the expected *NRCF* without fertilization (*N* rate is 0) and the expected *NRCF* at each nitrogen rate (*N*). The *EFB* values will show the increase in profit with respect to the increased fertilizer amount:

$$EFB(N) = NRCF(N) - NRCF(N=0) \quad (16)$$

where *EFB(N)* is the expected fertilization benefits ($/ha) at each nitrogen rate of N. *NRCF(N=0)* is the expected *NRCF* at the nitrogen application rate 0, and *NRCF(N)* is the *NRCF* at nitrogen amount *N* (kg/ha). Likewise, using this list of *NRCF* values, it is possible to find the economically optimum nitrogen rate (*EONR*), which is where the maximum *NRCF* is found from all considered application rates. Applying fertilizer beyond the proposed *EONR* may not lead to a further increase in net profits for the given agronomic conditions.

Results and Discussions

After completion of the numeric computation for a submitted request, the DSS sends a detailed report to the user as an e-mail message containing the results presented in different ways. The report includes a graph to visualize the estimated net returns of fertilization which provides the expected profit (*NRCF*) response as a function of nitrogen rate (Figure 5). A second graph allows the user to view the probabilities of different *NRCF* values at any given application rate, which corresponds to the computed profit space graph (Figure 6). In addition to these graphs, the report also includes the summary of *NRCF* and, *EFB* values at each potential nitrogen application rate. In the example illustrated in Figure 5 and Figure 6, medium values for all qualitative or quantitative input features were selected: clay loam soil, moderate weather (medium AWDR and CHU values) as well as medium nutrient contributor from a previous crop.

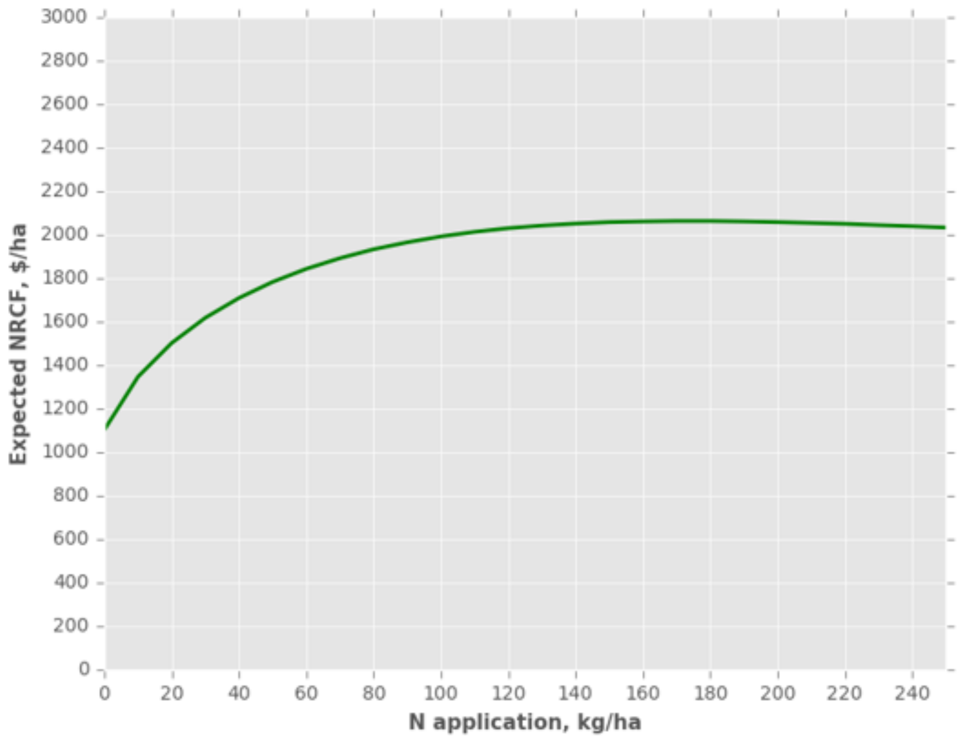

Figure 5. The expected NRCF as a function of fertilizer application rate

Analyzing the *NRCF* values predicted for every application rate is a traditional way to determine optimum application rates. However, to assess the financial risk affiliated with different management scenarios (Anton 2009), one can, for example, analyze the probability of profit to be above a certain threshold and select the rate related to the anticipated profit with an acceptable level of probability (Figure 6). The decline in probability at given combinations of NRCF and application rate corresponds to an increase of the associated risk for these combinations, and vice versa. In Figure 6, the risk was classified into four subjective categories based on ranges of probabilities such as certainly (between 75% and 100%), most likely (50% to 75%), and possibly (25% to 50%). Higher anticipated NRCF values lead to higher risks at a chosen application rate. However, higher application rates shift maximum probability towards higher NRCF, i.e., an increase of fertilization rates reduces the NRCF achievement risk for the selected profit range. The probability of NRCF reduction is higher when a lower application rate than EONR rate is accepted and vice versa. Based on the QP model restrictions, the probability of severe economic loss for over-application is rather small.

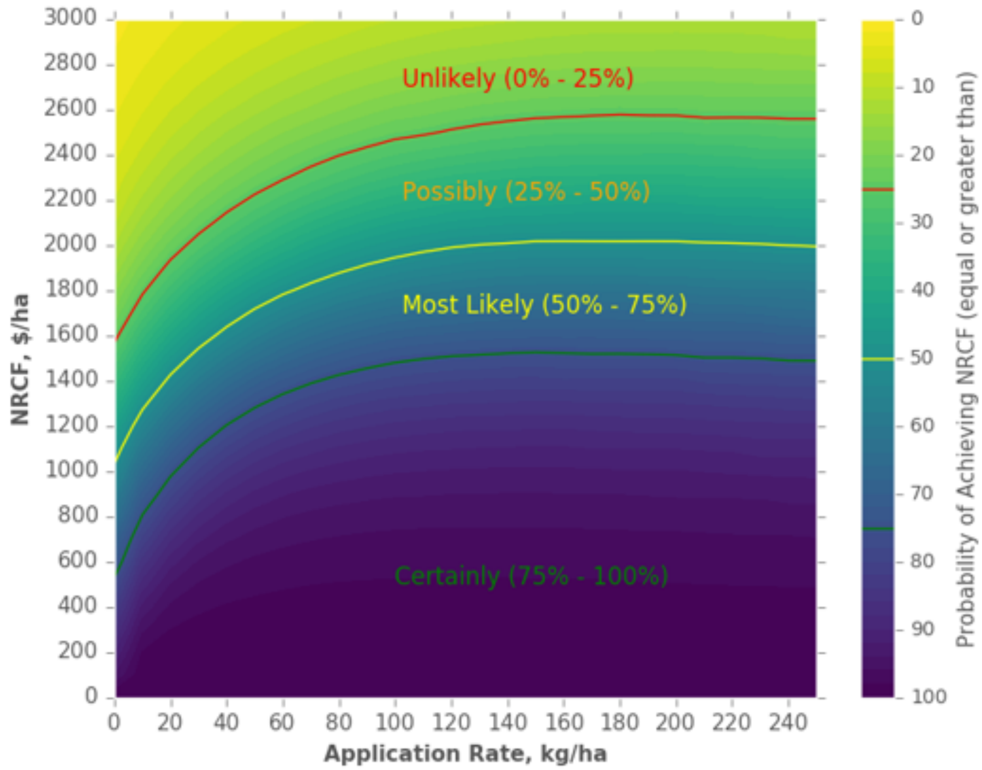

Figure 6. Profit space, profits and probabilities as a function of application rate.

The above analysis is based on current production conditions, and predictions are derived from these known conditions. However, the DSS can also be used to analyze the possible effects on expected returns of changes in the growing conditions over time and place. This can be performed through a sensitivity analysis of what happens when input features are changed.

Sensitivity analysis

The sensitivity of the DSS was examined by providing flexible scenarios varying the categories used in the user input form. For example, the sensitivity to precipitation was illustrated by alternatively selecting the dry, medium and wet conditions, while keeping constant the other attributes. The values for the constant attributes are listed in Figure 7, which shows the profit response for each precipitation category. Apparently, the medium conditions were found to be optimum and balanced in terms of profitable scenarios (Table 1). The reduction was lower with very wet conditions, whereas a significant economic loss was observed with very dry conditions. This indicates that the soil (loam) was more tolerant to abundant water than water stress. The DSS estimated near 40 $/ha higher profits for medium conditions when compared to very dry conditions at the optimum application rate. The EONR was the same for all the scenarios since the fertilizer cost was relatively low, and results may be less sensitive to the cost of fertilizer than to the price of yield.

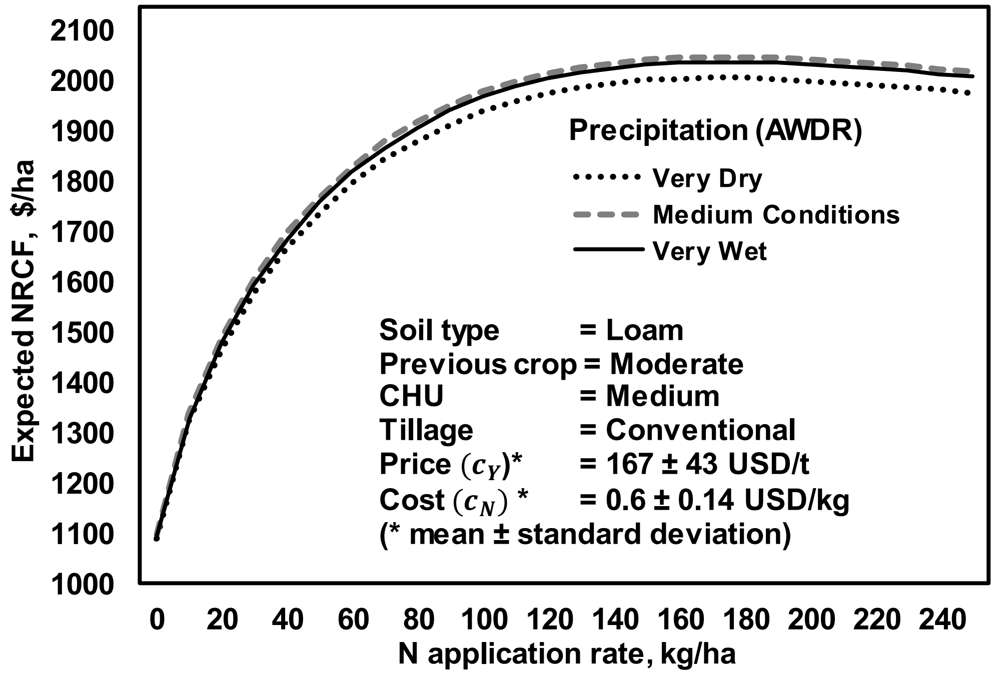

Figure 7. Profit response for the precipitation (AWDR) classes.

Table 1.Sensitivity assessment of climate conditions (precipitation) on the expected benefits.

AWDR	*EONR* kg/ha	*EFB* $/ha	*NRCF* $/ha	Probabilities of *NRCF* >			
				1000 $/ha	1500 $/ha	2000 $/ha	2500 $/ha
Very Dry	170	922	2008	91%	72%	47%	25%
Medium	170	955	2050	91%	75%	50%	27%
Very Wet	170	956	2040	91%	74%	49%	27%

Other analyses done with the DSS indicate that the sensitivity to precipitation would change under a different input/production context. For example, the economic benefits expected with the same precipitation categories would likely change if a clay soil was specified instead of a loam soil.

The sensitivity to fertilizer cost can be assessed by specifying the costs and prices structure manually in the user input. To illustrate sensitivity to fertilizer cost, independent scenarios were submitted in subsequent requests to the DSS, in which the mean fertilizer cost was high (1.2 $/kg), low (0.3 $/kg) and average (0.6 $/kg). **Error! Reference source not found.**, which includes the values for the features that were kept constant, illustrates the profit response lines for the different cost values.

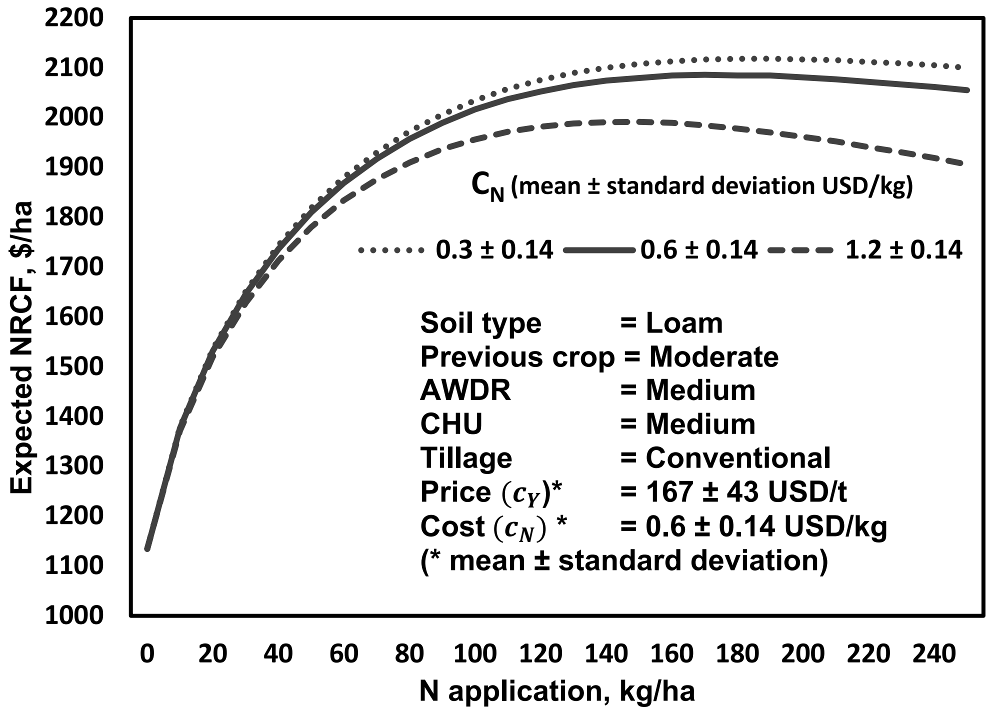

Figure 8. The expected profits for the fertilizer low, average and high-cost scenarios.

Apparently, if the cost was high, the system proposed lower application rates (EONR =150 kg/ha) and there was a decline in the expected profit of about 90 $/ha, and 20 kg/ha in the EONR when compared to the average cost scenario (Table 2). On the other hand, if the fertilizer was cheaper (low cost) then the system proposed very high application rates (EONR = 190 kg/ha, 20 kg/ha more than with average cost) with higher net returns than the other scenarios. This makes sense since increasing fertilizer costs lowers the potential profits, and vice versa. The probability of profit achievement was higher when the mean cost was low, and vice versa.

Table 2. Sensitivity assessment of fertilizer cost on the expected economic returns.

Cost value	Mean USD/kg	*EONR* kg/ha	*EFB* $/ha	*NRCF* $/ha	Probabilities of *NRCF* >			
					1000 $/ha	1500 $/ha	2000 $/ha	2500 $/ha
Low	0.3	190	983	2118	93%	78%	55%	31%
Average	0.6	170	952	2086	91%	76%	52%	29%
High	1.2	150	856	1991	90%	73%	48%	26%

Although a sensitivity analysis can be performed with the other features, only sensitivity to precipitation and fertilizer cost were shown here to illustrate the process. These sensitivity analyses have shown that it is possible to anticipate the variability in the economic returns caused by changes to the production context. The results were based on records currently available in the database and are expected to change with the addition of new trials. In future, with a more complete database, it should be possible to assess the impact of changes in management, such as going from conventional tillage to no-till practices.

Conclusion

The NumericAg DSS enables the determination of the optimum average application rate that maximizes expected profits (net return over cost of fertilization) for a given set of farm specific conditions. Probabilities associated with possible anticipated profits, at a given application rate, were found to be useful in determining an application rate based on the individual risk preference. The DSS handles production uncertainties by modeling the probability of yield prediction errors, by considering economic uncertainties and by using price and cost distributions. Uncertainty-based treatment of each model input and dynamic assessment of the importance of each record in the underlying production database, using the record similarity concept, constitute the unique features of this algorithm. The DSS also allows for a better understanding of the sensitivity of expected economic benefits for various input features.

The current version is illustrated using only one crop and one fertilizer disregarding effect of split application. However, the methods and equations specified in the material and methods section can be generalized and used with other fertilizer variants (such as potassium or phosphorus as a single controlled input), and with other crops provided the appropriate database of prior historical trials for the respective fertilizer and crop has been included in the database. The DSS is evolving with ongoing advancements and the prototype application can be accessed online at http://www.numericag.com.

Acknowledgments

This research was supported in part by Agriculture and Agri-Food Canada’s Agri Innovation Program and by Fertilizer Canada with the North American 4R Research Fund for the overarching project, “A Canadian Research Network to Improve 4R Nutrient Stewardship for Environmental Health and Crop Production"

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